GATE

Electronics Engineering

(Previous Years Solved Papers 1987-1994)

Contents

MADE EASY
PUBLICATIONS

Network Theory

UNIT I

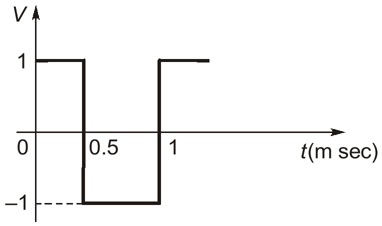
CONTENTS

I

Network Theory

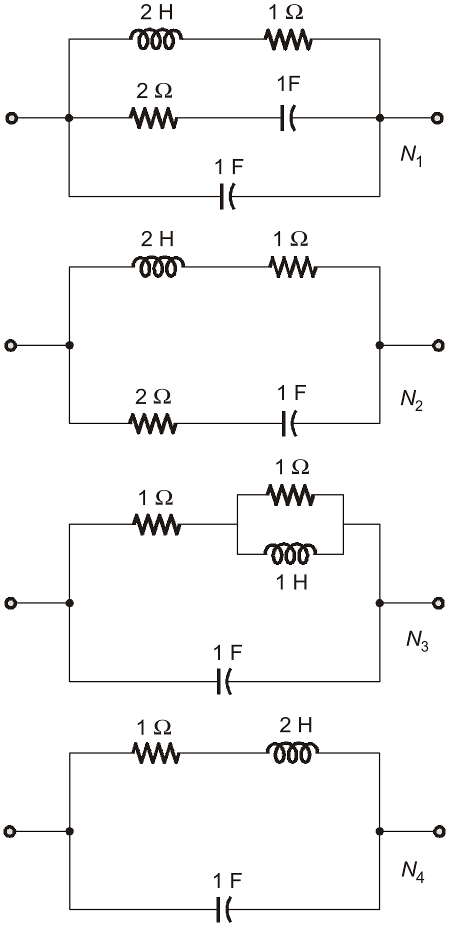
1. Basics of Network Analysis

1.1 A square waveform as shown in figure is applied across 1 mH ideal inductor. The current through the inductor is a .... wave of .... peak amplitude.



[1987 : 2 Marks]

1.2 Of the networks, N_1, N_2, N_3 and N_4 of figure, the networks having identical driving point function are



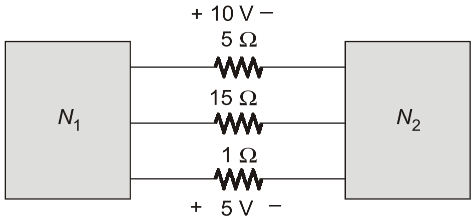
(a) N_1 and N_2 (b) N_2 and N_4
(c) N_1 and N_3 (d) N_1 and N_4

[1992 : 2 Marks]

1.3 A network contains linear resistors and ideal voltage sources. If values of all the resistors are doubled, then the voltage across each resistor is

(a) halved
(b) doubled
(c) increased by four times
(d) not changed [1993 : 2 Marks]

1.4 The two electrical subnetworks N_1 and N_2 are connected through three resistors as shown in Fig. The voltages across 5 ohm resistor and 1 ohm resistor are given to be 10 V and 5 V, respectively. Then voltage across 15 ohm resistor is



(a) – 105 V (b) + 105 V
(c) – 15 V (d) + 15 V

[1993 : 2 Marks]

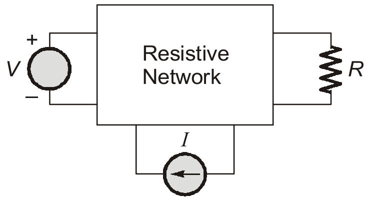
1.5 A dc circuit shown in figure has a voltage source *V*, a current source *I* and several resistors. A particular resistor *R* dissipates a power of 4 Watts when *V* alone is active. The same resistor *R* dissipates a power of 9 Watts when *I* alone is active. The power dissipated by *R* when both sources are active will be

(a) 1 W

(b) 5 W

(c) 13 W

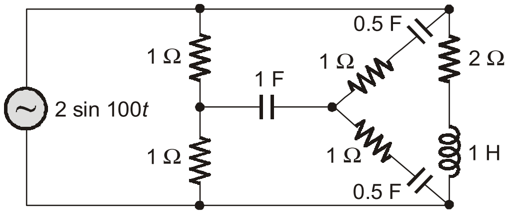
(d) 25 W



[1993 : 1 Mark]

2. Sinusoidal Steady State

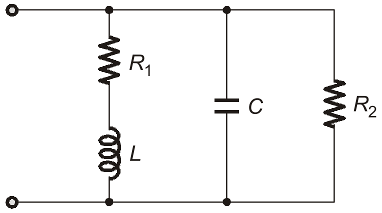
2.1 The value of current through the 1 Farad capacitor of figure is



(a) zero
(b) one
(c) two
(d) three

[1987 : 2 Marks]

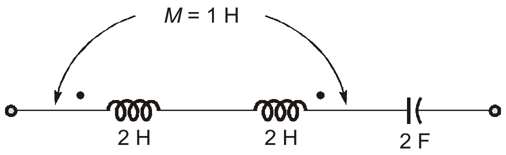
2.2 The half – power bandwidth of the resonant circuit of figure can be increased by:



(a) increasing R_1
(b) decreasing R_1
(c) increasing R_2
(d) decreasing R_2

[1989 : 2 Marks]

2.3 The resonant frequency of the series circuit shown in figure is



(a) $\frac{1}{4\pi\sqrt{3}}$Hz
(b) $\frac{1}{4\pi}$Hz
(c) $\frac{1}{2\pi\sqrt{10}}$Hz
(d) $\frac{1}{4\pi\sqrt{2}}$Hz

[1990 : 2 Marks]

2.4 In a series RLC high *Q* circuit, the current peaks at a frequency

(a) equal to the resonant frequency
(b) greater than the resonant frequency
(c) less than the resonant frequency
(d) none of the above is true

[1991 : 2 Marks]

2.5 For the series *R-L-C* circuit of fig. 1, the partial phasor diagram at a certain frequency is a shown in fig. 2. The operating frequency of the circuit is

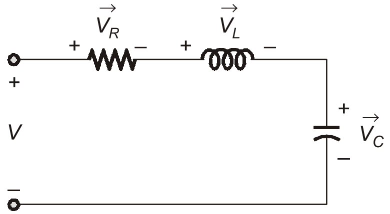


Figure-1

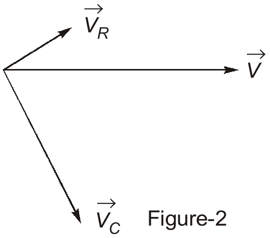
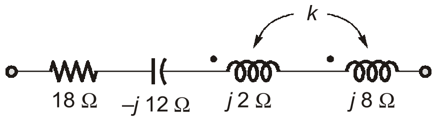


Figure-2

(a) equal to the resonance frequency
(b) less than the resonance frequency
(c) greater than the resonance frequency
(d) not zero

[1992 : 2 Marks]

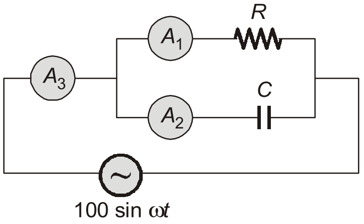
2.6 In the series circuit shown in figure, for series resonance, the value of the coupling coefficient *k* will be



(a) 0.25
(b) 0.5
(c) 0.999
(d) 1.0

[1993 : 1 Mark]

2.7 In figure, A_1, A_2 and A_3 are ideal ammeters. If A_1 reads 5 A, A_2 reads 12 A, then A_3 should read



(a) 7 A
(b) 12 A
(c) 13 A
(d) 17 A

[1993 : 2 Marks]

2.8 A series LCR circuit consisting of $R = 10\ \Omega$, $|X_L| = 20\ \Omega$ and $|X_C| = 20\ \Omega$ is connected across an a.c. supply of 200 V rms. The rms voltage across the capacitor is

(a) $200 \angle -90°$ V
(b) $200 \angle +90°$ V
(c) $400 \angle +90°$ V
(d) $400 \angle -90°$ V

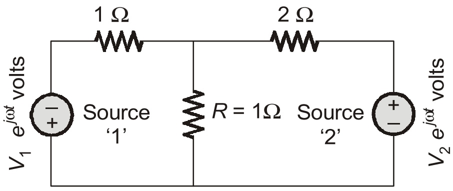
[1994 : 1 Mark]

3. Network Theorems

3.1 If an impedance Z_L is connected across a voltage source V with source impedance Z_S, then for maximum power transfer the load impedance must be equal to

(a) source impedance Z_S
(b) complex conjugate of Z_S
(c) real part of Z_S
(d) imaginary part of Z_S [1988 : 2 Marks]

3.2 In the circuit of figure, the power dissipated in the resistor R is 1 W when only source '1' is present and '2' is replaced by a short. The power dissipated in the same resistor R is 4 W when only source '2' is present and '1' is replaced by a short. When both the sources '1' and '2' are present, the power dissipated in R will be :

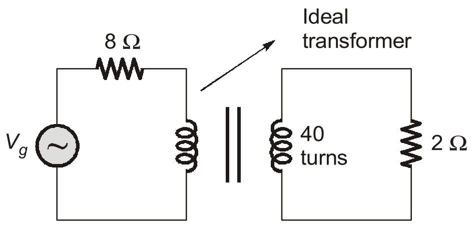


(a) 1 W (b) 3 W
(c) 4 W (d) 5 W
[1989 : 2 Marks]

3.3 A load, $Z_L = R_L + jX_L$ is to be matched, using an ideal transformer, to a generator of internal impedance, $Z_S = R_S + jX_S$. The turns ratio of the transformer required is

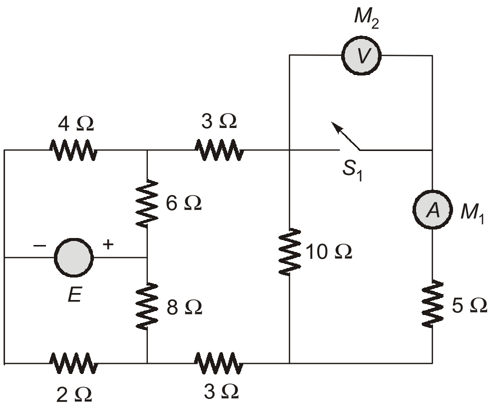
(a) $\sqrt{|Z_L / Z_S|}$ (b) $\sqrt{|R_L / R_S|}$
(c) $\sqrt{|R_L / Z_S|}$ (d) $\sqrt{|Z_L / R_S|}$
[1989 : 2 Marks]

3.4 If the secondary winding of the ideal transformer shown in the circuit of figure has 40 turns, the number of turns in the primary winding for maximum power transfer to the 2 Ω resistor will be



(a) 20 (b) 40
(c) 80 (d) 160 [1993 : 1 Mark]

3.5 In the circuit of figure, when switch S_1 is closed, the ideal ammeter M_1 reads 5 A. What will the ideal voltmeter M_2 read when S_1 is kept open? (The value of E is not specified).



[1993 : 2 Marks]

3.6 A generator of internal impedance, Z_G, delivers maximum power to a load impedance, Z_L, only if Z_L = ...

[1994 : 1 Mark]

4. Transient Analysis

4.1 A 10 Ω resistor, a 1 H inductor and 1 μF capacitor are connected in parallel. The combination is driven by a unit step current. Under the steady state condition, the source current flows through :

(a) the resistor
(b) the inductor
(c) the capacitor only
(d) all the three elements [1989 : 2 Marks]

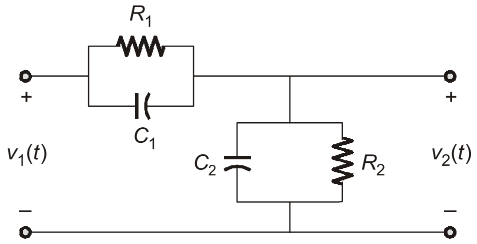
4.2 If the Laplace transform of the voltage across a capacitor of value of $\frac{1}{2}$F is

$$V_C(s) = \frac{s+1}{s^3 + s^2 + s + 1},$$

the value of the current through the capacitor at $t = 0^+$ is

(a) 0 A (b) 2 A
(c) (1/2) A (d) 1 A
[1989 : 2 Marks]

4.3 For the compensated attenuator of figure, the impulse response under the condition $R_1C_1 = R_2C_2$ is



(a) $\frac{R_2}{R_1+R_2}[1-e^{\frac{1}{R_1C_1}}]u(t)$

(b) $\frac{R_2}{R_1+R_2}\delta(t)$

(c) $\frac{R_2}{R_1+R_2}u(t)$

(d) $\frac{R_2}{R_1+R_2}e^{\frac{t}{R_1C_1}}\,u(t)$ **[1992 : 2 Marks]**

4.4 A ramp voltage, $v(t) = 100\,t$ Volts, is applied to an RC differentiating circuit with R = 5 kΩ and C = 4 μF. The maximum output voltage is

(a) 0.2 volt (b) 2.0 volts
(c) 10.0 volts (d) 50.0 volts

[1994 : 1 Mark]

5. Two Port Networks

5.1 Two two-port networks are connected in parallel. The combination is to be represented as a single two-port network. The parameters of this network are obtained by addition of the individual

(a) z parameters
(b) h parameters
(c) y parameters
(d) ABCD parameters

[1988 : 2 Marks]

5.2 For the transfer function of a physical two-port network:

(a) all the zeros must lie only in the left half of the s-plane
(b) the poles may lie anywhere in the s-plane
(c) the poles lying on the imaginary axis must be simple
(d) a pole may lie at origin

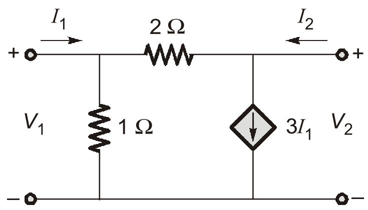
[1989 : 2 Marks]

5.3 The condition $AD - BC = 1$ for a two-port network implies that the network is a:

(a) reciprocal network
(b) lumped element network
(c) lossless network
(d) unilateral element network

[1989 : 2 Marks]

5.4 The open circuit impedance matrix of the two port network shown in figure is



(a) $\begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & -8 \\ -8 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

[1990 : 2 Marks]

5.5 Two Two-port networks are connected in cascade. The combination is to the represented as a single two-port network. The parameters of the network are obtained by multiplying the individual

(a) z-parameter matrices
(b) h-parameter matrices
(c) y-parameter matrices
(d) ABCD parameter matrices

[1991 : 2 Marks]

5.6 For a 2-port network to be reciprocal,

(a) $z_{11} = z_{22}$ (b) $y_{21} = y_{12}$
(c) $h_{21} = -h_{12}$ (d) $AD - BC = 0$

[1992 : 2 Marks]

5.7 The condition, that a 2-port network is reciprocal, can be expressed in terms of its $ABCD$ parameters as ____.

[1994 : 1 Mark]

6. Graph Theory and State Equations

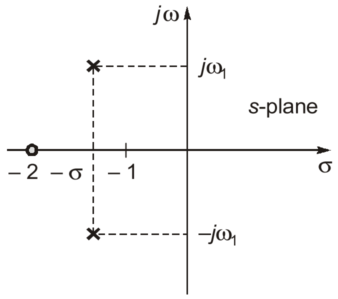
6.1 Relative to a given fixed tree of a network,

(a) Link currents form an independent set.
(b) Branch currents form an independent set.
(c) Link voltages form an independent set.
(d) Branch voltages form an independent.

[1992 : 2 Marks]

7. Network Functions

7.1 A driving point admittance function has pole and zero locations as shown below. The range of s for which the function can be realized using passive elements is



(a) $\sigma < -1$ (b) $\sigma > 1$
(c) $\sigma < 1$ (d) $\sigma > -1$

[1988 : 2 Marks]

7.2 The necessary and sufficient condition for a rational function of s, $T(s)$ to be a driving point impedance of an RC network is that all poles and zeros should be

(a) Simple and lie on the negative real axis of the s-plane
(b) Complex and lie in the left half of the s-plane
(c) Complex and lie in the right half of the s-plane
(d) Simple and lie on the positive real axis of the s-plane

[1991 : 2 Marks]

7.3 Indicate True/False and give reason (For the following question)

$Z(s) = \dfrac{5}{s^2+4}$ represents the input impedance of a network.

[1994 : 2 Marks]

■■■

Answers Network Theory

1.1	(0.5)	1.2	(c)	1.3	(d)	1.4	(a)	1.5	(d)	2.1	(a)	2.2	(a) & (d)
2.3	(b)	2.4	(a)	2.5	(b)	2.6	(a)	2.7	(c)	2.8	(d)	3.1	(b)
3.2	(a)	3.3	(a)	3.4	(c)	3.5	(5)	3.6	Sol.	4.1	(b)	4.2	(c)
4.3	(b)	4.4	(b)	5.1	(c)	5.2	(c & d)	5.3	(a)	5.4	(a)	5.5	(d)
5.6	(b & c)	5.7	(1)	6.1	(a)	7.1	(b)	7.2	(a)	7.3	(False)		

Explanations Network Theory

1. Basics of Network Analysis

1.1 (0.5)

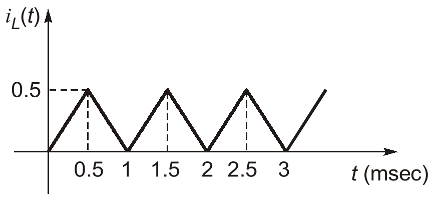
Triangular wave, 0.5 Amp peak.

$$i_L = \frac{1}{L}\int V dt$$

So, the current through inductor is the integration of the applied voltage across the inductor.
Integration of square wave is a triangular wave.
So, the current through the inductor is a ***triangular wave***.

Now, $v(t) = u(t) - 2u(t-0.5) + 2u(t-1) + \ldots$

$\therefore$ $i_L(t) = r(t) - 2r(t-0.5) + 2r(t-1) + \ldots$



$I_p = 0.5$ A

1.2 (c)

$$N_1: \quad Y_1(s) = s + \frac{1}{2s+1} + \frac{1}{\frac{1}{s}+2}$$

$$Y_1(s) = \frac{2s^2+2s+1}{2s+1}$$

N_2: $$Y_2(s) = \frac{1}{2s+1} + \frac{1}{2+\frac{1}{s}} = \frac{1+s}{2s+1}$$

N_3: $$Y_3(s) = s + \frac{1}{1+\frac{1}{1+\frac{1}{s}}} = s + \frac{1+s}{s+1+s}$$

$$Y_3(s) = \frac{2s^2+2s+1}{2s+1}$$

N_4:

$$Y_4(s) = s + \frac{1}{2s+1} = \frac{2s^2+s+1}{2s+1}$$

So, N_1 and N_3 networks having identical driving point function.

1.3 (d)

If all resistors are doubled then the current get halved.

$$I' = \frac{I}{2}$$

$$R' = 2R$$

$$V' = \frac{I}{2} \cdot 2R = IR = V$$

1.4 (a)

Current through 5 Ω resistor,

$$i_5 = \frac{10}{5} = 2 \text{ Amp.}$$

Current through 1 Ω resistor,

$$i_1 = \frac{5}{1} = 5 \text{ Amp.}$$

So, the current through 15 Ω resistor,

$$i_{15} = -(i_1 + i_5) = -(5+2) = -7 \text{ Amp.}$$

Voltage across 15 Ω resistor.

$$V_{15} = 15(i_{15}) = 15(-7) = -105 \text{ V}$$

1.5 (d)

$$P_1 = 4 \text{ W and } P_2 = 9 \text{ W}$$

From superposition theorem

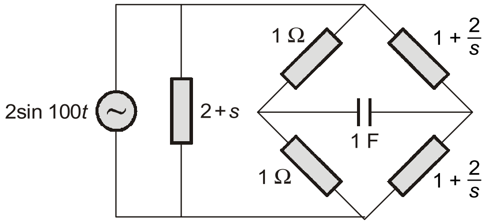
$$P = (\sqrt{P_1} + \sqrt{P_2})^2 = (\sqrt{4} + \sqrt{9})^2$$

$$P = (2+3)^2 = 25 \text{ W}$$

2. Sinusoidal Steady State

2.1 (a)

The given circuit is a bridge,



Product of opposite arms are equal,

$$1\left(1+\frac{2}{s}\right) = 1\left(1+\frac{2}{s}\right)$$

So, the current through the diagonal element (1 F capacitor) is zero.

2.2 (a) & (d)

Selectivity ∝ Q

$$Q = \frac{f_r}{B.W.} \quad ; \quad Q \propto \frac{1}{B.W.}$$

$$B.W. \propto \frac{1}{Q} \quad ; \quad B.W. \propto \frac{1}{\text{selectivity}}$$

If $R_1 \to 0$

and $R_2 \to \infty$

then the circuit will have only L & C elements and has high selectivity.

So, the half power bandwidth can be increased by reducing the selectivity.

So, by increasing the series resistance R_1 and decreasing the parallel resistance R_2, the half power bandwidth can be increased.

2.3 (b)

$$L_{eq} = L_1 + L_2 - 2M$$

$$L_{eq} = 2 + 2 - 2(1) = 2 \text{ H}$$

At resonance

$$X_L = X_C$$

$$\omega L_{eq} = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{L_{eq} C}$$

$$\omega = \frac{1}{\sqrt{2 \times 2}} = \frac{1}{2} \text{ rad/sec}$$

$$2\pi f = \frac{1}{2}$$

$$f = \frac{1}{4\pi} \text{Hz}$$

2.4 **(a)**

At resonance frequency

$$Z_{\min} = R$$

$$I_{\max} = \frac{V}{Z_{\min}}$$

2.5 **(b)**

Given network is the series *R-L-C* circuit in resistor *R*, voltage V_R and current I_R is in phase and in series circuit current is same in all the elements.

$$I = I_R$$

So, the current is leading the voltage in the circuit. So, the given circuit will behave as capacitive circuit.

$$V_C > V_L$$

$$IX_C > IX_L$$

$$X_C > X_L$$

$$\frac{1}{\omega C} > \omega L$$

$$\omega^2 < \frac{1}{LC}$$

$$\omega^2 < \omega_r^2$$

$$\omega < \omega_r$$

2.6 **(a)**

At resonance

$$X_L - X_C = 0$$

$$|X_L| = |X_C|$$

$$|X_L| = |j12|$$

$$X_L = j12$$

$$X_L = X_{L_1} + X_{L_2} + 2k\sqrt{X_{L_1} \cdot X_{L_2}}$$

$$X_L = j2 + j8 + 2k\sqrt{j2\,j8} = j12$$

$$2\,k\,j4 = j2$$

$$k = 0.25$$

2.7 **(c)**

$$A_3^2 = A_1^2 + A_2^2$$

$$A_3^2 = (5)^2 + (12)^2$$

$$A_3^2 = 169$$

$$A_3 = 13 \text{ Amp.}$$

2.8 **(d)**

$\because \; X_L = X_C$

So, the circuit is at resonance.

$$I = \frac{V}{R} = \frac{200}{10} = 20 \text{ Amp.}$$

Voltage across the capacitor

$$V_c = I(-jX_c) = 20(-j20) = -j400$$

$$V_c = 400\angle{-90^\circ} \text{ V}$$

3. Network Theorems

3.1 **(b)**

According to maximum power transfer theorem,

$$Z_L = Z_S^*$$

3.2 **(a)**

$$P_1 = 1 \text{ W} \quad ; \quad P_2 = 4 \text{ W}$$

Since the polarity of both the sources are different

$$P = (\sqrt{P_1} - \sqrt{P_2})^2$$

$$P = (\sqrt{1} - \sqrt{4})^2 = (1-2)^2 = 1 \text{ W}$$

3.3 **(a)**

$$\frac{Z_L}{Z_S} = \left(\frac{n_2}{n_1}\right)^2$$

$$\frac{n_2}{n_1} = \sqrt{\left|\frac{Z_L}{Z_S}\right|}$$

3.4 **(c)**

$$\left(\frac{n_2}{n_1}\right)^2 = \frac{|Z_L|}{|Z_S|}$$

$$\left(\frac{n_2}{n_1}\right)^2 = \frac{2}{8} = \frac{1}{4}$$

$$\frac{n_2}{n_1} = \frac{1}{2}$$

$$n_1 = 2n_2 = 2 \times 40 = 80$$

3.5 **(5)**

Across switch S_1,

$$I_{sc} = 5 \text{ A}$$

$$R_{\text{Th}} = \left[(4\|6 + 2\|8) + 3 + 3\right]\|10 + 5$$

$$R_{\text{Th}} = (2.4 + 1.6 + 3 + 3) \,\|\, 10 + 5$$

$$= 10 \,\|\, 10 + 5 = 5 + 5$$

$$R_{\text{Th}} = 10\ \Omega$$

$$V_{o/c} = V_{AB} = I_{sc} R_{\text{Th}} = 5 \times 10 = 50 \text{ V}$$

3.6 **Sol.**

$$Z_L = Z_G^*$$

$$Z_G = R_G + jX_G$$

$$Z_L = R_G - jX_G$$

4. Transient Analysis

4.1 (b)

At steady state

Inductor behave as short circuit.

So, under steady state condition the source current flows through the inductor.

4.2 (c)

$$Z_C(s) = \frac{1}{Cs} = \frac{2}{s}$$

$$I_C(s) = \frac{V_C(s)}{Z_C(s)} = \frac{s(s+1)}{2(s^3+s^2+s+1)} = \frac{s(s+1)}{2(s^2+1)(s+1)}$$

$$I_C(s) = \frac{s}{2(s^2+1)}$$

$$i(0^+) = \lim_{s\to\infty} s\,I_C(s) = \lim_{s\to\infty} \frac{s^2}{2(s^2+1)}$$

$$= \frac{1}{2+0} = \frac{1}{2}\text{ Amp.}$$

4.3 (b)

$$Z_2(s) = \frac{R_2 \times \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} = \frac{R_2}{R_2 C_2 s + 1}$$

$$Z_1(s) = \frac{R_1 \times \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} = \frac{R_1}{R_1 C_1 s + 1}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$R_1 C_1 = R_2 C_2$$

$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{R_2}{R_2 C_2 s + 1}}{\frac{R_1}{R_2 C_2 s + 1} + \frac{R_2}{R_2 C_2 s + 1}} = \frac{R_2}{R_1 + R_2}$$

$$V_2(s) = \frac{R_2}{R_1 + R_2} V_1(s)$$

For impulse response.

$$V_1(s) = 1$$

$$v_1(t) = \delta(t)$$

$$v_2(t) = \frac{R_2}{R_1 + R_2}\delta(t)$$

4.4 (b)

$$V_o = RC\frac{dV_i}{dt} = (5\times 10^3)(4\times 10^{-6})\frac{d}{dt}(100t)$$

$$V_o = 2\text{ Volts}$$

5. Two Port Networks

5.1 (c)

$$[Y] = [Y]_A + [Y]_B$$

5.2 (c) & (d)

The poles lying on the imaginary axis must be simple.

A pole may lie at origin.

5.3 (a)

For reciprocal network,

$AD - BC = 1$

5.4 (a)

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{12} = \frac{1\times I_2}{I_2} = 1\,\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$Z_{22} = \frac{2I_2 + 1I_2}{I_2} = 3\,\Omega$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{11} = -\frac{2I_1\times 1}{I_1} = -2$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$Z_{21} = \frac{-6I_1 + V_1}{I_1} = \frac{-6I_1 - 2I_1}{I_1}$$

$$Z_{21} = -8$$

5.5 (d)

ABCD parameter matrices,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

5.6 (b) & (c)

$$y_{21} = y_{12} \quad ; \quad h_{21} = -h_{12}$$

5.7 **(1)**

$AD - BC = 1$

7. Network Functions

7.1 **(b)**

$$\sigma - 1 > 0$$
$$\sigma > 1$$

7.2 **(a)**

Simple and lie on the negative real axis of the *s*-plane.

The poles and zeros of the $Z_{RC}(s)$ should be simple and alternate on the negative real axis of the *s*-plane.

7.3 **(False)**

For $Z(s)$ to represent the input impedance of a passive network, the numerator and denominator degrees should not differ by more than 1.

So, $Z(s) = \dfrac{5}{s^2 + 4}$ can not represent the input impedance of a network **(False)**.

■■■

Electromagnetics

UNIT II

CONTENTS

II

Electromagnetics

1. Basics of Electromagnetics

1.1 An electrostatic field is said to be conservative when:

(a) The divergence of the field is equal to zero

(b) The curl of the field is equal to zero

(c) The curl of the field is equal to $-\frac{\partial E}{\partial t^2}$

(d) The Laplacian of the field is equal to $\mu\varepsilon\frac{\partial^2 E}{\partial t^2}$

[1987 : 2 Marks]

1.2 On either side of a charge - free interface between two media

(a) the normal components of the electric field are equal

(b) the tangential component of the electric field are equal

(c) the normal components of the electric flux density are equal

(d) the tangential components of the electric flux density are equal

[1988 : 2 Marks]

1.3 Vector potential is a vector

(a) whose curl is equal to the magnetic flux density

(b) whose curl is equal to the electric field intensity

(c) whose divergence is equal to the electric potential

(d) which is equal to the vector product $E \times H$

[1988 : 2 Marks]

1.4 The electric field strength at a far - off point P due to a point charge, $+q$ located at the origin, O is 100 millivolts/metre. The point charge is now enclosed by a perfectly conducting hollow metal sphere with its centre at the origin, O. The electric field strength at the point, P

(a) remains unchanged in its magnitude and direction

(b) remains unchanged in its magnitude but reverse in direction

(c) would be that due to a dipole formed by the charge, $+q$, at O and $-q$ induced

(d) would be zero

[1989 : 2 Marks]

1.5 Which of the following field equations indicate that the free magnetic charge do not exist

(a) $\vec{H} = \frac{1}{\mu}\nabla \times \vec{A}$ (b) $\vec{H} = \oint \frac{I\,\vec{dl} \times \vec{R}}{4\pi R^2}$

(c) $\nabla \cdot \vec{H} = 0$ (d) $\nabla \times \vec{H} = \vec{J}$

[1990 : 2 Marks]

1.6 Given $\vec{V} = x\cos^2 y\hat{i} + x^2 e^z \hat{j} + z\sin^2 y\hat{k}$ and S the surface of unit cube with one corner at the origin and edges parallel to the coordinate axis, the value of the integral $\iint_C \vec{V}.\hat{n}\,dS$ is ______.

[1993 : 2 Marks]

1.7 For a uniformly charged sphere of radius R and charge density ρ, the ratio of magnitude of electric fields at distances $R/2$ and $2R$ from the centre,

i.e., $\frac{E(r = R/2)}{E(r = 2R)}$ is _____

[1993 : 2 Marks]

1.8 Match **List-I** with **List-II** and select the correct answer using the code given below the Lists:

List-I	List-II
A. $\nabla \times \vec{H} = \vec{J}$	1. Continuity equation
B. $\oint_C \vec{E} \cdot \vec{d}l = -\oint_s \frac{\partial B}{\partial t} \cdot \vec{ds}$	2. Faraday's Law
C. $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$	3. Ampere's Law
	4. Gauss's Law
	5. Biot-Savart Law

Codes:

	A	B	C
(a)	3	2	1
(b)	2	1	3
(c)	4	3	1
(d)	1	2	3

[1994 : 2 Marks]

2. Uniform Plane Waves

2.1 For an electromagnetic wave incident from one medium to a second medium, total reflection takes place when
(a) The angle of incidence is equal to the Brewster angle with *E* field perpendicular to the plane of incidence
(b) The angle of incidence is equal to the Brewster angle with *E* field parallel to the plane of incidence
(c) The angle of incidence is equal to the critical angle with the wave moving from the denser medium to a rarer medium
(d) The angle of incidence is equal to the critical angle with the wave moving from a rarer medium to a denser medium

[1987 : 2 Marks]

2.2 In a good conductor the phase relation between the tangential components of electric field E_t and the magnetic field H_t is as follows
(a) E_t and H_t are in phase
(b) E_t and H_t are out of phase
(c) H_t leads E_t by 90°
(d) E_t leads H_t by 45°

[1988 : 2 Marks]

2.3 The skin depth of copper at a frequency of 3 GHz is 1 micron (10^{-6} metre). At 12 GHz, for a non magnetic conductor whose conductivity is 1/9 times that of copper, the skin depth would be
(a) $\sqrt{9 \times 4}$ microns (b) $\sqrt{9/4}$ microns
(c) $\sqrt{4/9}$ microns (d) $1/\sqrt{9 \times 4}$ microns

[1989 : 2 Marks]

2.4 The incoming solar radiation at a place on the surface of the earth is 1.2 kW/m^2. The amplitude of the electric field corresponding to this incident power is nearly equal to
(a) 80 mV/m (b) 2.5 V/m
(c) 30 V/m (d) 950 V/m

[1990 : 2 Marks]

2.5 The electric field component of a uniform plane electromagnetic wave propagating in the *Y*-direction in a lossless medium will satisfy the equation

(a) $\dfrac{\partial^2 E_y}{\partial y^2} = \mu \in \dfrac{\partial^2 E_y}{\partial t^2}$ (b) $\dfrac{\partial^2 E_y}{\partial x^2} = \mu \in \dfrac{\partial^2 E_y}{\partial t^2}$

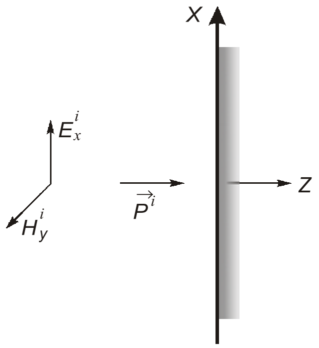
(c) $\dfrac{\partial^2 E_x}{\partial y^2} = \mu \in \dfrac{\partial^2 E_x}{\partial t^2}$ (d) $\dfrac{\sqrt{E_x^2 + E_z^2}}{\sqrt{H_x^2 + H_z^2}} = \sqrt{\mu / \in}$

[1991 : 2 Marks]

2.6 A material is described by the following electrical parameters at a frequency of 10 GHz: $\sigma = 10^6$ mho/m, $\mu = \mu_0$ and $\in / \in_0 = 10$. The material at this frequency is considered to be ($\in_0 = \dfrac{1}{36\pi} \times 10^{-9}$ F/m)
(a) a good conductor
(b) a good dielectric
(c) neither a good conductor, nor a good dielectric
(d) a good magnetic material

[1993 : 2 Marks]

2.7 A plane wave is incident normally on a perfect conductor as shown in figure. Here E_x^i, H_y^i and $\vec{P}^i$ are electric field, magnetic field and Poynting vector, respectively, for the incident wave. The reflected waver should have



(a) $E_x^r = -E_x^i$ (b) $H_y^r = -H_y^i$
(c) $\vec{P}^r = -\vec{P}^i$ (d) $E_x^r = E_x^i$

[1993 : 2 Marks]

2.8 A long solenoid of radius *R*, and having *N* turns per unit length carries a time dependent current $I(t) = I_0 \cos(\omega t)$. The magnitude of induced electric field at a distance *R*/2 radially from the axis of the solenoid is

(a) $\frac{R}{2}\mu_o N I_0 \omega \sin(\omega t)$

(b) $\frac{R}{4}\mu_o N I_0 \omega \cos(\omega t)$

(c) $\frac{R}{4}\mu_o N I_0 \omega \sin(\omega t)$

(d) $R\mu_o N I_0 \omega \sin(\omega t)$

[1993 : 2 Marks]

2.9 $\oint_C \vec{A}.\vec{dl} = \int_S \_\_\_.\vec{ds}$ [1994 : 1 Mark]

2.10 A plane electromagnetic wave traveling along the $+z$ direction, has its electric field given by $E_x = 2\cos(\omega t)$ and $E_y = 2\cos(\omega t + 90)°$ the wave is

(a) linearly polarized
(b) right circularly polarized
(c) left circularly polarized
(d) elliptically polarized [1994 : 1 Mark]

3. Transmission Lines

3.1 A transmission line of real characteristic impedance is terminated with an unknown load. The measured value of VSWR on the line is equal to 2 and a voltage minimum point is found to be at the load. The load impedance is then

(a) Complex (b) Purely capacitive
(c) Purely resistive (d) Purely inductive

[1987 : 2 Marks]

3.2 A Two-wire transmission line of characteristic impedance Z_0 is connected to a load of impedance $Z_L (Z_L \neq Z_0)$. Impedance matching cannot be achieved with

(a) a quarter - wavelength transformer
(b) a half - wavelength transformer
(c) an open - circuited parallel stub
(d) a short-circuited parallel stub

[1988 : 2 Marks]

3.3 A 50 ohm lossless transmission line has a pure reactance of (j 100) ohms as its load. The VSWR in the line is

(a) 1/2 (Half) (b) 2 (Two)
(c) 4 (Four) (d) ∞ (Infinity)

[1989 : 2 Marks]

3.4 The input impedance of a short circuited lossless transmission line quarter wave long is

(a) purely reactive
(b) purely resistive
(c) infinite
(d) dependent on the characteristic impedance of the line

[1991 : 2 Marks]

3.5 A transmission line whose characteristic impedance is a real

(a) must be a lossless line
(b) must be a distortionless line
(c) may not be a lossless line
(d) may not be a distortionless line

[1992 : 2 Marks]

3.6 Consider a transmission line of characteristic impedance 50 ohm. Let it be terminated at one end by ($+j$ 50) ohm. The VSWR produced by it in the transmission line will be

(a) +1 (b) 0
(c) ∞ (d) $+j$

[1993 : 2 Marks]

3.7 A load impedance, $(200 + j0)\ \Omega$ is to be matched to a 50 Ω lossless transmission line by using a quarter wave line transformer (QWT). The characteristic impedance of the QWT required is ....

[1994 : 1 Mark]

3.8 If a pure resistance load, when connected to a lossless 75 ohm line, produces a VSWR of 3 on the line, then the load impedance can only be 25 ohms. **True/False (Give Reason)**

[1994 : 2 Marks]

4. Waveguides

4.1 The cut off frequency of a waveguide depends upon

(a) The dimensions of waveguide
(b) The dielectric property of the medium in the waveguide

(c) The characteristic impedance of the waveguide
(d) The transverse and axial components of the fields
[1987 : 2 Marks]

4.2 For a normal mode *EM* wave propagating in a hollow rectangular wave guide
(a) the phase velocity is greater than the group velocity.
(b) the phase velocity is greater than velocity of light in free space.
(c) the phase velocity is less than the velocity of light in free space.
(d) the phase velocity may be either greater than or less than the group velocity.
[1988 : 2 Marks]

4.3 Choose the correct statements
For a wave propagating in an air filled rectangular waveguide
(a) Guided wavelength is never less than the free space wavelength.
(b) Wave impedance is never less than the free space impedance.
(c) Phase velocity is never less than the free space velocity.
(d) *TEM* mode is possible if the dimensions of the wave guide are properly chosen.
[1990 : 2 Marks]

4.4 The interior of a $\frac{20}{3}\text{cm} \times \frac{20}{4}\text{cm}$ rectangular waveguide is completely filled with a dielectric of $\in_r = 4$. Waves of free space wave lengths shorter than ..... can be propagated in the TE_{11} mode.
[1994 : 1 Mark]

5. Antennas

5.1 The electric field *E* and the magnetic field *H* of a short dipole antenna satisfy the condition
(a) the *r* component of *E* is equal to zero
(b) both *r* and θ components of *H* are equal to zero
(c) the θ component of *E* dominates the *r* component in the far - field region
(d) the θ and ϕ components of *H* are of the same order of magnitude in the near field region
[1988 : 2 Marks]

5.2 Two isotropic antennas are separated by a distance of two wavelengths. If both the antennas are fed with currents of equal phase and magnitude, the number of lobes in the radiation pattern in the horizontal plane are
(a) 2 (b) 4
(c) 6 (d) 8
[1990 : 2 Marks]

5.3 In a broad side array of 20 isotropic radiators, equally spaced at distance of $\lambda/2$, the beam width between first nulls is
(a) 51.3 degrees (b) 11.46 degrees
(c) 22.9 degrees (d) 102.6 degrees
[1991 : 2 Marks]

5.4 Two dissimilar antennas having their maximum directivities equal, which of the following statements are right?
(a) must have their beam widths also equal
(b) cannot have their beam widths equal because they are dissimilar antenna
(c) may not necessarily have their maximum power gains equal
(d) must have their effective aperture areas (capture areas) also equal
[1992 : 2 Marks]

5.5 The beamwidth between first nulls of a uniform linear array of *N* equally spaced (element spacing = *d*), equally excited antennas, is determine by
(a) *N* alone and not by *d*
(b) *d* alone and not by *N*
(c) the ratio, (*N*/*d*)
(d) the product, (*Nd*)
[1992 : 2 Marks]

5.6 For a half-wave dipole antenna, which of the following statements are right?
(a) the radiation intensity is maximum along the normal to the dipole axis
(b) the current distribution along its length is uniform irrespective of the length
(c) the effective length equals its physical length
(d) the input impedance is independent of the location of the feed-point
[1994 : 1 Mark]

■■■

Answers Electromagnetics

1.1	(b)	1.2	(b & c)	1.3	(a)	1.4	(a)	1.5	(c)	1.6	(1)	1.7	(2)
1.8	(a)	2.1	(c)	2.2	(d)	2.3	(b)	2.4	(d)	2.5	(c & d)	2.6	(a)
2.7	(a & c)	2.8	(c)	2.9	(sol.)	2.10	(c)	3.1	(c)	3.2	(b)	3.3	(d)
3.4	(c)	3.5	(b & c)	3.6	(c)	3.7	(100)	3.8	(False)	4.1	(a, b)	4.2	(a & b)
4.3	(a & c)	4.4	(8)	5.1	(b, c)	5.2	(d)	5.3	(b)	5.4	(a, c)	5.5	(d)
5.6	(a)												

Explanations Electromagnetics

1. Basics of Electromagnetics

1.1 (b)

An electrostatic field is said to be conservative when the closed line integral of the field is zero.

$$\oint \vec{E}\cdot\vec{dl} = 0$$

Stoke's theorem

$$\oint_c \vec{E}\cdot\vec{dl} = \int_s (\nabla\times\vec{E})\cdot\vec{ds}$$

So, $\nabla\times\vec{E} = 0$

1.2 (b) & (c)

Boundary conditions: $E_{t_1} = E_{t_2}$ and $D_{N_1} = D_{N_2}$

1.3 (a)

Vector potential in magnetic fields is the measure of work done to move a current element $I\vec{dl}$

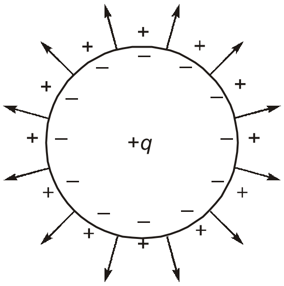
$$\vec{A} = \frac{W}{I\vec{dl}}$$

It relates to $\vec{B}$ (which is force experienced by a current element) as $\nabla\times A = \vec{B}$.

$$\nabla\times\vec{A} = \vec{B} = \mu\vec{H}$$

1.4 (a)

The radially outward field is normal to conductor and no induction effects E is same.



1.5 (c)

Magnetic field lines are closed around the current and have no sources or sink points.

$$\oiint B\cdot ds = \iint \nabla\cdot B\,dv$$
$$\nabla\cdot B = 0$$

In any closed surface,

Entering flux = Leaving flux

Dipole is the cause of magnetic fields.

Magnetic monopoles or free charges do not exist is justified by $\nabla\cdot B = 0$.

According to the Gauss's law for magnetic fields.

$$\oint_s \vec{B}\cdot\vec{ds} = 0$$
$$\nabla\cdot\vec{B} = 0$$
$$\nabla\cdot\mu\vec{H} = 0$$
$$\mu\nabla\cdot\vec{H} = 0$$
$$\nabla\cdot\vec{H} = 0$$

1.6 (1)

$$\oiint \vec{V}\cdot\vec{ds} = \iiint (\nabla\cdot\vec{V})\,dv$$
$$\nabla\cdot\vec{V} = \cos^2 y + \sin^2 y = 1$$
$$\int (\nabla\cdot\vec{V})\,dv = 1$$

1.7 (2)

$$D = \frac{\rho_v r}{3} \qquad r < R$$

$$D = \frac{\rho_v R^3}{3r^2} \qquad r > R$$

Applying Gauss law with spherical Gaussian surface concentric with the charge.

$$E(\text{at } r = R/2) = \frac{\rho_v R}{6\in}$$

$$E(2R) = \frac{\rho_v R}{12 \in}$$

So, $\dfrac{E(at\ r = R/2)}{E(at\ r = 2R)} = 2$

1.8 **(a)**

(A) - 3, (B) - 2, (C) - 1

$$\nabla \times \vec{H} = \vec{J} \rightarrow \text{Ampere's law}$$

$$\oint_c \vec{E} \cdot \vec{dl} = -\oint_s \frac{\partial B}{\partial t} \cdot \vec{ds} \rightarrow \text{Faraday's law}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{dt} \rightarrow \text{Continuity equation}$$

2. Uniform Plane Waves

2.1 **(c)**

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\sqrt{\frac{\in_{r_2}}{\in_{r_1}}}$$

For total internal reflection,

$$\theta_i \geq \theta_c$$

$$\theta_i \geq \sin^{-1}\left(\sqrt{\frac{\in_{r_2}}{\in_{r_1}}}\right) \text{ with } \in_{r_1} > \in_{r_2}$$

For total internal reflection to take place, the wave should move from a denser medium to a rarer medium and the angle of incidence should be greater than or equal to the critical angle.

2.2 **(d)**

η = intrinsic impedance

$$= \frac{E_t}{H_t} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega \in}}$$

For a good conductor

$$\sigma >> \omega\in$$

$$\frac{E_t}{H_t} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\pi/4}$$

So, E_t leads H_t by an angle of 45°.

2.3 **(b)**

We know that: For a good conductor,

$$\text{Skin depth} = \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\delta \propto \frac{1}{\sqrt{f\sigma}}$$

$$\frac{\delta_2}{\delta_1} = \sqrt{\frac{f_1 \sigma_1}{f_2 \sigma_2}}$$

Given that,

$$\delta_1 = 1 \text{ micron}$$

$$f_1 = 3 \text{ GHz}$$

$$f_2 = 12 \text{ GHz} \qquad \frac{\sigma_2}{\sigma_1} = \frac{1}{9}$$

$$\frac{\delta_2}{1} = \sqrt{\frac{3}{12} \times \frac{9}{1}} = \sqrt{\frac{9}{4}}$$

$$\delta_2 = \sqrt{\frac{9}{4}} \text{ microns}$$

2.4 **(d)**

Power density, $P = \dfrac{E^2}{2\eta}$

$$E = \sqrt{2\eta P}$$

$$\eta = \eta_o = 120\pi$$

$$P = 1.2 \text{ kW/m}^2 = 1200 \text{ W/m}^2$$

$$E = \sqrt{2 \times (120\pi) \times (1200)}$$

$$E \cong 950 \text{ V/m}$$

2.5 **(c & d)**

When E is a function of y and directed in x and z possibly orthogonal to propagation.

$$E_y = H_y = 0$$

$$\frac{\partial^2 E_x}{\partial y^2} = \mu \in \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{E}{H} = \frac{\sqrt{E_x^2 + H_z^2}}{\sqrt{H_x^2 + H_z^2}} = \eta = \sqrt{\frac{\mu}{\in}}$$

2.6 **(a)**

Loss tangent,

$$\frac{\sigma}{\omega \in} = \frac{10^6}{2\pi(10 \times 10^9)\,(8.854 \times 10^{-12} \times 10)}$$

$$\frac{\sigma}{\omega \in} = 1.798 \times 10^5 >> 1$$

So, the given material at f = 10 GHz is considered as a good conductor.

2.7 **(a & c)**

Reflection coefficient for electric fields

$$\Gamma_E = \frac{\sqrt{\dfrac{j\omega\mu}{\sigma}} - 120\pi}{\sqrt{\dfrac{j\omega\mu}{\sigma}} + 120\pi} = -1 \text{ for conductors}$$

As $$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} \simeq 0 \text{ with } \sigma \text{ being large}$$

when E reflection completely out of phase, H reflects in phase

$$E_x^r = -E_x^i$$
$$H_x^r = H_x^i$$

So, $$\vec{P}^r = -\vec{P}^i$$

2.8 (c)

$$H \cdot l = nI(t)$$

where, n = Total number of turns

$$N = \frac{n}{l}$$
$$H = NI(t)$$
$$B = \mu_o(H) = \mu_o NI(t)$$
$$B = \mu_o NI_o \cos(\omega t)$$

According to Maxwell's equation,

$$\oint E \cdot \vec{dl} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$
$$E \cdot 2\pi \frac{R}{2} = \mu_o NI_o \omega \sin(\omega t) \frac{\pi R^2}{4}$$
$$E = \frac{R}{4} \mu_o NI_o \omega \sin(\omega t)$$

2.9 Sol.

Using Stoke's theorem

$$\oint_C \vec{A} \cdot \vec{dl} = \int_S \nabla \times \vec{A} \cdot \vec{ds}$$

2.10 (c)

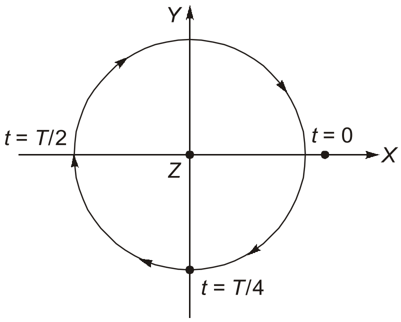
Two equal E field components out of phase by 90° gives circular polarization.

$$E_x = 2\cos\omega t$$
$$E_y = 2\cos(\omega t + 90°) = -2\sin\omega t$$

Tracing with time for sense of rotation as left or right.

$t = 0$ $E_x = 2$ $E_y = 0$

$t = \frac{T}{4}$ $E_x = 0$ $E_y = -2$



Left circular for $+z$ propagation.

3. Transmission Lines

3.1 (c)

Given, voltage minimum (V_{min}) point is at load.

If V_{min} or V_{max} occurs at the load for a lossless transmission line then load impedance Z_L is purely resistive.

If Z_L (resistive) > Z_o, voltage maxima occurs at the load.

If Z_L (resistive) < Z_o, voltage minima occurs at the load.

3.2 (b)

$\lambda/2$ length transmission line has $Z_{in} = Z_L$ and hence cannot be a impedance matching device.

3.3 (d)

$$\text{Reflection coefficient} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$= \frac{j100 - 50}{j100 + 50}$$
$$|\Gamma| = \frac{\sqrt{(100)^2 + (50)^2}}{\sqrt{(100)^2 + (50)^2}} = 1$$
$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1+1}{1-1} = \frac{2}{0} = \infty$$

3.4 (c)

For a quarter wave transformer ($\lambda/4$),

$$Z_{in} = \frac{Z_0^2}{Z_L}$$
$$Z_L = 0$$
$$Z_{in} = \frac{Z_0^2}{0} = \infty$$

3.5 (b & c)

For a distortionless line,

$$RC = LG$$
$$\alpha = \sqrt{RG}$$
$$\beta = \omega\sqrt{LC}$$
$$Z_o = R_o = \sqrt{\frac{L}{C}} \text{ (real)}$$

For a lossless line,

$$R = 0$$
$$G = 0$$
$$\alpha = 0$$
$$\beta = \omega\sqrt{LC}$$
$$Z_o = R_o = \sqrt{\frac{L}{C}} \text{ (real)}$$

Z_o is real means either lossless or distortionless and lossless lines also satisfies $LG = RC$ which means they are distortionless.

3.6 (c)

$$\text{Reflection coefficient} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{j50 - 50}{j50 + 50} = \frac{-50 + j50}{50 + j50}$$

$$|\Gamma| = \frac{\sqrt{(50)^2 + (50)^2}}{\sqrt{(50)^2 + (50)^2}} = 1$$

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1+1}{1-1} = \frac{2}{0} = \infty$$

3.7 (100)

For quarter wave line transformer,

$$Z_0^2 = Z_{in} Z_L$$
$$Z_0^2 = (50)(200)$$
$$Z_0 = 100\ \Omega$$

3.8 (False)

For a pure resistive load of a lossless line,

$$\text{VSWR} = \frac{R_L}{R_0} \quad \text{if} \quad R_L > R_0$$

$$\text{VSWR} = \frac{R_0}{R_L} \quad \text{if} \quad R_0 > R_L$$

Given that, VSWR = 3

So, $R_L = \text{VSWR} \times R_0 = 3 \times 75 = 225\ \Omega$

$$\text{or} \quad R_L = \frac{R_0}{\text{VSWR}} = \frac{75}{3} = 25\ \Omega$$

Hence, $R_L = 25$ or 75

4. Waveguides

4.1 (a, b)

$$f_c = \frac{v_o}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Where, $v_o = \dfrac{c}{\sqrt{\mu_r \epsilon_r}}$

$$f_c = \frac{c}{\sqrt{\mu_r \epsilon_r} \times 2}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

So, f_c depends on 'a' and 'b' (dimensions of waveguide)

$$f_c \propto \frac{1}{\sqrt{\mu_r \epsilon_r}}$$

So, f_c depends upon the dielectric property of the medium in the waveguide.

4.2 (a & b)

$$v_p > c > v_g$$

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$v_g = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

4.3 (a & c)

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g > \lambda$$

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\eta_{TE} > \eta_0$$

$$\eta_{TM} = \eta_0\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\eta_{TM} < \eta_0$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r \epsilon_r}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$v_p > c$$

4.4 **(8)**

$$v = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_0 \in_0 \in_r}}$$

$$= \frac{1}{\sqrt{\mu_0 \in_0}} \times \frac{1}{\sqrt{\in_r}}$$

$$v = \frac{3\times 10^8}{\sqrt{4}}$$

$$= 1.5\times 10^8 \text{ m/sec}$$

$$m = n = 1$$

$$f_c = \frac{v}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$= \frac{1.5\times 10^8}{2}\sqrt{\left(\frac{3}{20}\right)^2 + \left(\frac{4}{20}\right)^2} \times 10^2$$

$$f_c = \frac{7.5\times 10^9}{20}\times 5 = 1.875\times 10^9 \text{ Hz}$$

$$\lambda_c = \frac{v}{f_c} = \frac{1.5\times 10^8}{1.875\times 10^9} = 8 \text{ cm}$$

So, waves of free space wavelength shorter than 8 cm can be propagated.

$$\lambda_c = 8 \text{ cm}$$

5. Antennas

5.1 **(b, c)**

Every dipole antenna has fields in directions of r, θ, ϕ with E_r, E_θ, H_ϕ.

These fields depend on $1/r$, $1/r^2$ and $1/r^3$ with distance.

$1/r$ terms are called as radiation fields and exist even at far away distance.

E_θ and $H\phi$ have 1 terms each depending on $1/r$.

$1/r^2$ and $1/r^3$ terms are called as induction fields and exist at closer distances with zero values far away.

E_r has only $1/r^2$ and $1/r^3$ terms.

Option (a)-wrong-E_r exists.

Option (b)-right-E_ϕ only exists.

Option (c)-right-E_θ has $1/r$ terms which E_r does not have.

Option (d)-wrong-H_θ does not exist.

5.2 **(d)**

Given that: $d = 2\lambda$ and $\alpha = 0$

$$\psi = \alpha + \beta d \cos\theta$$

$$\psi = 0 + \frac{2\pi}{\lambda}\cdot 2\lambda \cdot \cos\theta = 4\pi\cos\theta$$

When $\psi = 0, 2\pi, -2\pi, 4\pi, -4\pi$ maxima occurs in that direction.

Maximum at $\theta = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

So, number of lobes in the radiation pattern in the horizontal plane = 8.

5.3 **(b)**

Electric field strength of N element array

$$E_T = NE_o \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

For null points $\sin\left(N\frac{\psi}{2}\right) = 0$

with $\sin\left(\frac{\psi}{2}\right) \neq 0$ (denominator term)

$$\left(N\frac{\psi}{2}\right) = 2n\pi \text{ for null points}$$

$$\psi = \frac{4n\pi}{N}$$

$$\alpha + \beta d\cos\theta = \frac{4n\pi}{N}$$

With $\alpha = 0$ for broadside array.

$$\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\theta = \frac{4n\pi}{N}$$

$$\cos\theta = \frac{4n}{N}$$

with $n = 1$ for 1st null points

$$\cos\theta_{NP1} = \frac{4}{20}$$

$$\theta_{NP1} = 78.46$$

with $n = 2$ for 2nd null points

$$\cos\theta_{NP2} = \frac{8}{20}$$

$$\theta_{NP2} = 66.42$$

Beam width between first nulls = 12.04°

5.4 **(a, c)**

$$\text{Directivity} = \frac{4\pi}{\lambda^2}A_e = \frac{4\pi}{\Omega_A}$$

Ω_A = Beam solid angle

A_e = Capture area

Power gain depends on efficiency and losses, so they may not be equal is right.

Capture areas depend on gain and frequency also.

5.5 (d)

Beam widths depend on the array factor $\frac{N\psi}{2}$.

when $\frac{N\psi}{2} = 2n\pi$ for minima condition

$$\alpha + \beta d\cos\theta = \psi = \frac{4n\pi}{N}$$

$$\cos\theta = \frac{\frac{4n\pi}{N} - \alpha}{\beta d} = \frac{4n\pi - \alpha N}{\beta N d}$$

$\alpha = 0$ for equally excited (in phase)

$$\cos\theta \propto \frac{1}{Nd}$$

BWFN in both arrays is determined by the product (Nd).

5.6 (a)

- Dipole antenna has radiation depending as $\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$ and maximum at 90° to array axis.
- Current distribution is uniform only for very short lengths like hertzian dipole.
- $l_{\text{eff}} = \frac{2l}{\pi}$ for $\lambda/2$ dipole
- Z_{in} for any wire antenna depends on loading effects from either sides of the feed points or length on either sides.

■■■

Control Systems

UNIT III

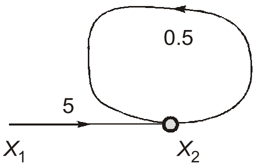
CONTENTS

III

Control Systems

1. Basics of Control Systems, Block Diagram and SFG's

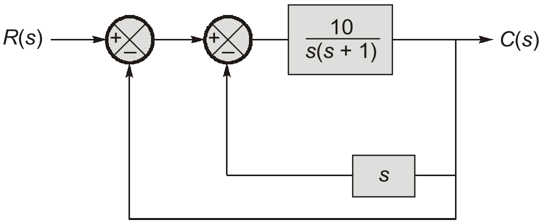
1.1 In the signal flow graph shown in figure $X_2 = TX_1$ where T, is equal to



(a) 2.5 (b) 5
(c) 5.5 (d) 10

[1987 : 2 Marks]

1.2 For the system shown in figure the transfer function $\frac{C(s)}{R(s)}$ is equal to :

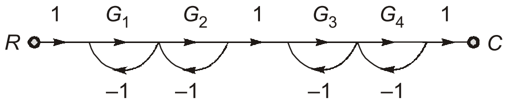


(a) $\frac{10}{s^2+s+10}$ (b) $\frac{10}{s^2+11s+10}$

(c) $\frac{10}{s^2+9s+10}$ (d) $\frac{10}{s^2+2s+10}$

[1987 : 2 Marks]

1.3 The C/R for the signal flow graph in figure is:



(a) $\frac{G_1G_2G_3G_4}{(1+G_1G_2)(1+G_3G_4)}$

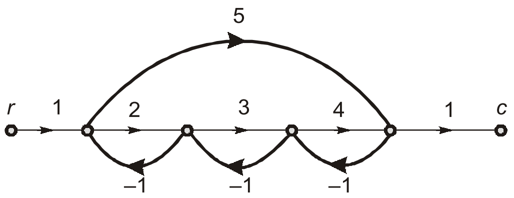
(b) $\frac{G_1G_2G_3G_4}{(1+G_1+G_2+G_1G_2)(1+G_3+G_4+G_3G_4)}$

(c) $\frac{G_1G_2G_3G_4}{(1+G_1+G_2)(1+G_3+G_4)}$

(d) $\frac{G_1G_2G_3G_4}{(1+G_1+G_2+G_3+G_4)}$

[1989 : 2 Marks]

1.4 In the signal flow graph of figure the gain c/r will be



(a) 11/9 (b) 22/15
(c) 24/23 (d) 44/23

[1991 : 2 Marks]

2. Compensators and Controllers

2.1 The transfer function of a simple RC network functioning as a controller is:

$$G_c(s) = \frac{s+z_1}{s+p_1}$$

The condition for the RC network to act as a phase lead controller is:

(a) $p_1 < z_1$ (b) $p_1 = 0$
(c) $p_1 = z_1$ (d) $p_1 > z_1$

[1990 : 2 Marks]

2.2 A process with open-loop model $G(s) = \frac{Ke^{-sT_D}}{\tau s+1}$ is controlled by a PID controller. For this process.

(a) the integral mode improves transient performance
(b) the integral mode improves steady-state performance
(c) the derivative mode improve transient performance
(d) the derivative mode improves steady-state performance.

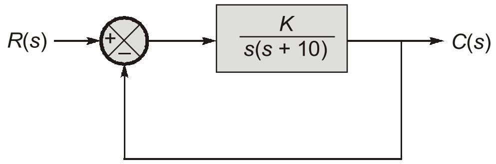
[1992 : 2 Marks]

2.3 Tachometer feedback in a d.c. position control system enhances stability **(T/F)**

[1994 : 1 Mark]

3. Time Response Analysis

3.1 The unity feedback system shown in fig. has:



(a) Zero steady state position error

(b) Zero steady state velocity error

(c) Steady state position error $\frac{K}{10}$ units

(d) Steady state velocity error $\frac{K}{10}$ units

[1987 : 2 Marks]

3.2 The steady state error of a stable 'type 0' unity feedback system for a unit step function is

(a) 0 (b) $\frac{1}{1+K_P}$

(c) ∞ (d) $\frac{1}{K_P}$

[1990 : 2 Marks]

3.3 A second-order system has a transfer function given by $G(s) = \frac{25}{s^2+8s+25}$

If the system, initially at rest, is subjected to a unit step input at $t = 0$, the second peak in the response will occur at

(a) π sec (b) $\pi/3$ sec

(c) $2\pi/3$ sec (d) $\pi/2$ sec.

[1991 : 2 Marks]

3.4 A unity-feedback control system has the open-loop transfer function $G(s) = \frac{4(1+2s)}{s^2(s+2)}$ if the input to the system is a unit ramp, the steady-state error will be

(a) 0 (b) 0.5

(c) 2 (d) Infinity

[1991 : 2 Marks]

3.5 The poles of a continuous time oscillator are ......

[1994 : 1 Mark]

3.6 The response of an LCR circuit to a step input is

(a) Over damped

(b) Critically damped

(c) Oscillatory

If the transfer function has

(1) poles on the negative real axis

(2) poles on the imaginary axis

(3) multiple poles on the positive real axis

(4) poles on the positive real axis

(5) Multiple poles on the negative real axis.

[1994 : 2 Marks]

3.7 Match the following codes with List-I with List-II:

List-I

(a) Very low response at very high frequencies

(b) Over shoot

(c) Synchro-control transformer output

List-II

(1) Low pass systems

(2) Velocity damping

(3) Natural frequency

(4) Phase-sensitive modulation

(5) Damping ratio.

[1994 : 2 Marks]

4. Stability Analysis

4.1 Consider a characteristic equation given by

$$s^4 + 3s^3 + 5s^2 + 6s + K + 10$$

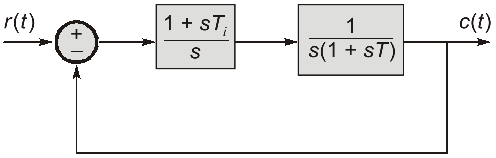
The condition for stability is

(a) $K > 5$ (b) $-10 < K$

(c) $K > -4$ (d) $-10 < K < -4$

[1988 : 2 Marks]

4.2 In order to stabilize the system shown in figure T_i should satisfy,



(a) $T_i = -T$ (b) $T_i = T$

(c) $T_i < T$ (d) $T_i > T$

[1989 : 2 Marks]

4.3 An electromechanical closed-loop control system has the following characteristic equation; $s^3 + 6K\,s^2 + (K + 2)s + 8 = 0$. Where K is the forward gain of the system. The condition for closed loop stability is:

(a) $K = 0.528$ (b) $K = 2$

(c) $K = 0$ (d) $K = -2.258$

[1990 : 2 Marks]

4.4 If $s^3 + 3s^2 + 4s + A = 0$, then all the roots of this equation are in the left half plane provided that

(a) $A > 12$ (b) $-3 < A < 4$

(c) $0 < A < 12$ (d) $5 < A < 12$

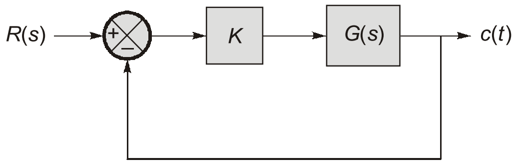
[1993 : 2 Marks]

4.5 If $G(s)$ is a stable transfer function, then $F(s) = \dfrac{1}{G(s)}$ is always a stable transfer function. **(T/F).**

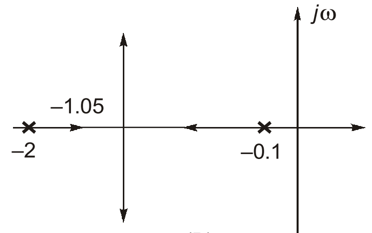
[1994 : 2 Marks]

5. Root Locus

5.1 Consider a closed-loop system shown in Figure (A) below. The root locus for it is shown in Figure (B). the closed loop transfer function for the system is



(A)



(B)

(a) $\dfrac{K}{1 + (0.5s + 1)(10s + 1)}$

(b) $\dfrac{K}{(s + 2)(s + 0.1)}$

(c) $\dfrac{K}{1 + K(0.5s + 1)(10s + 1)}$

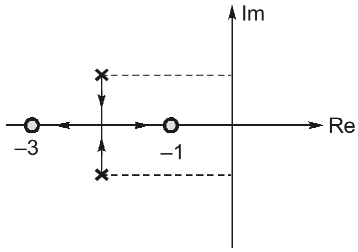
(d) $\dfrac{K}{K + 0.2(0.5s + 1)(10s + 1)}$ [1988 : 2 Marks]

5.2 The OLTF of a feedback system is

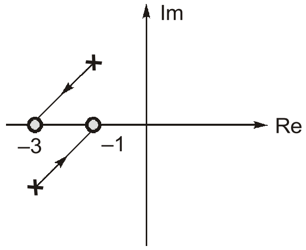
$$G(s)\,H(s) = \frac{K(s+1)(s+3)}{s^2 + 4s + 8}.$$

The root locus for the same is

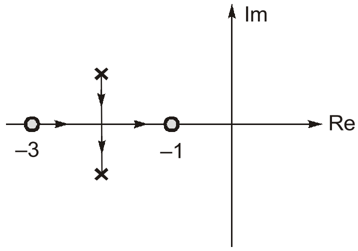
(a)



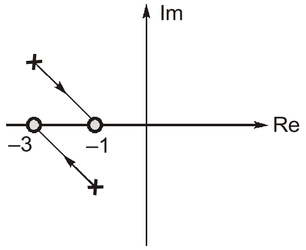
(b)



(c)



(d)



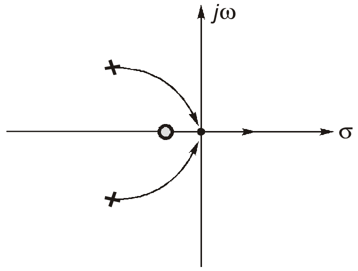
[1989 : 2 Marks]

5.3 The transfer function of a closed loop system is:

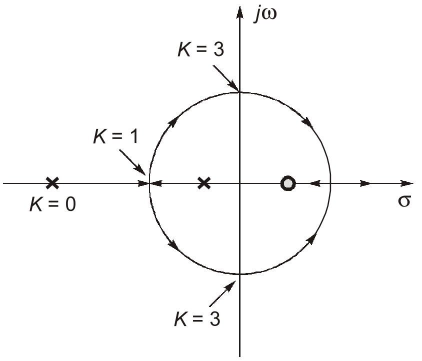
$$T(s) = \frac{K}{s^2 + (3 - K)(s + 1)}$$

Where K is the forward path gain. The root locus plot of the system is:

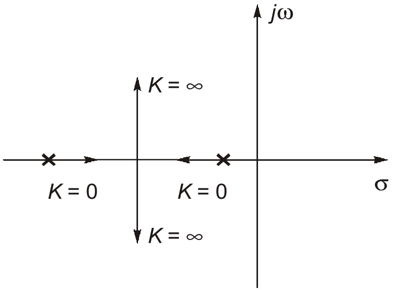
(a)



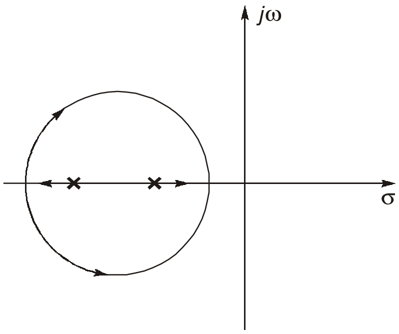
(b)



(c)



(d)



[1990 : 2 Marks]

5.4 The characteristic equation of a feedback control system is given by $s^3 + 5s^2 + (K + 6)s + K = 0$ Where $K > 0$ is a scalar variable parameter. In the root-locus diagram of the system the asymptotes of the root-loci for large values of K meet at a point in the s-plane, whose coordinates are

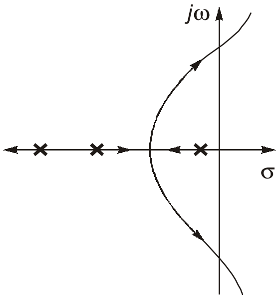
(a) (–3, 0) (b) (–2, 0)
(c) (–1, 0) (d) (2, 0)

[1991 : 2 Marks]

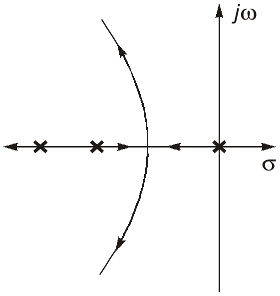
5.5 Given a unity feedback system with open-loop transfer function.

$G(s) = \dfrac{K}{s(s+1)(s+2)}$. The root locus plot of the system is of the form.

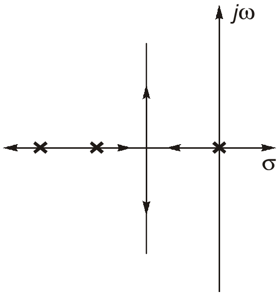
(a)



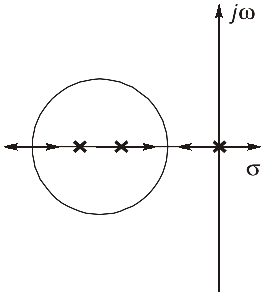
(b)



(c)



(d)



[1992 : 2 Marks]

5.6 If the open-loop transfer function is a ratio of a numerator polynomial of degree 'm' and a denominator polynomial of degree 'n' then the integer (n-m) represents the number of

(a) breakaway points
(b) unstable poles
(c) separate root loci
(d) asymptotes

[1994 : 1 Mark]

6. Frequency Response Analysis

6.1 A system has fourteen poles and two zeros. Its high frequency asymptote in its magnitude plot having a slope of:

(a) –40 dB/decade (b) –240 dB/decade
(c) –280 dB/decade (d) –320 dB/decade

[1987 : 2 Marks]

6.2 The polar plot of $G(s) = \frac{10}{s(s+1)^2}$ intercepts real axis at $\omega = \omega_0$. Then, the real part and ω_0 are respectively given by:

(a) –2.5, 1 (b) –5, 0.5
(c) –5, 1 (d) –5, 2

[1987 : 2 Marks]

6.3 The open-loop transfer function of a feedback control system is

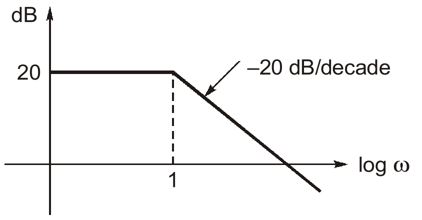
$$G(s) \cdot H(s) = \frac{1}{(s+1)^3}$$

The gain margin of the system is

(a) 2 (b) 4
(c) 8 (d) 16

[1991 : 2 Marks]

6.4 Bode plot of a stable system is shown in figure The transfer function of the system is



[1992 : 2 Marks]

6.5 The 3-dB bandwidth of a typical second-order system with the transfer function

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \text{ is given by}$$

(a) $\omega_n\sqrt{1-2\xi^2}$

(b) $\omega_n\sqrt{(1-\xi^2)+\sqrt{\xi^4-\xi^2+1}}$

(c) $\omega_n\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}$

(d) $\omega_n\sqrt{(1-2\xi^2)-\sqrt{4\xi^4-4\xi^2+2}}$

[1994 : 1 Mark]

6.6 The open loop frequency response of a system at two particular frequencies are given by : 1.2 ∠–180° and 1.0 ∠–190°. The closed loop unity feedback control is then ______

[1994 : 1 Mark]

7. State Space Analysis

7.1 Given the following state-space description of a system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Find the state-transition matrix.

[1988 : 2 Marks]

7.2 A linear second-order single-input continuous-time system is described by following set of differential equations

$\dot{x}_1(t) = -2x_1(t) + 4x_2(t)$;

$\dot{x}_2(t) = 2x_1(t) - x_2(t) + u(t)$

Where $x_1(t)$ and $x_2(t)$ are the state variables and $u(t)$ is the control variable. The system is

(a) controllable and stable
(b) controllable but unstable
(c) uncontrollable and unstable
(d) uncontrollable but stable. [1991 : 2 Marks]

7.3 A linear time-invariant system is described by the state variable model

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u.$$

$$Y = \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(a) The system is completely controllable
(b) The system is not completely controllable
(c) The system is completely observable
(d) The system is not completely observable

[1992 : 2 Marks]

■■■

Answers Control Systems

1.1	(d)	1.2	(b)	1.3	(c)	1.4	(d)	2.1	(d)	2.2	(b, c)	2.3	(True)
3.1	(a)	3.2	(b)	3.3	(a)	3.4	(a)	3.5	(sol.)	3.6	(sol.)	3.7	(sol.)
4.1	(d)	4.2	(d)	4.3	(b)	4.4	(c)	4.5	(False)	5.1	(d)	5.2	(a)
5.3	(a)	5.4	(b)	5.5	(a)	5.6	(d)	6.1	(b)	6.2	(c)	6.3	(c)
6.4	(sol.)	6.5	(c)	6.6	(–10)	7.1	Sol.	7.2	(b)	7.3	(b, c)		

Explanations Control Systems

1. Basics of Control Systems, Block Diagram and SFG's

1.1 (d)

$$\frac{X_2}{X_1} = \frac{5}{\Delta} = \frac{5}{1-(0.5)} = \frac{5}{0.5} = 10$$

1.2 (b)

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\dfrac{10}{s(s+1)}}{1+\dfrac{10}{s(s+1)}\times s}}{1+\dfrac{\dfrac{10}{s(s+1)}}{1+\dfrac{10}{s(s+1)}\times s}\times 1}$$

$$\frac{C(s)}{R(s)} = \frac{\dfrac{10}{s(s+1)+10s}}{1+\dfrac{10}{s(s+1)+10s}} = \frac{10}{s^2+s+10s+10}$$

$$\frac{C(s)}{R(s)} = \frac{10}{s^2+11s+10}$$

1.3 (c)

By using Masson's gain formulae

$$\frac{C}{R} = \frac{P_K\,\Delta_K}{\Delta}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{1-[-G_1-G_2-G_3-G_4] + [G_1G_3+G_1G_4+G_2G_3+G_2G_4]}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{1+G_1+G_2+G_3+G_4+G_1G_3 + G_1G_4+G_2G_3+G_2G_4}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{(1+G_1+G_2)(1+G_3+G_4)}$$

1.4 (d)

By using Masson's gain formulae,

$$\frac{C}{R} = \frac{\Sigma P_K\,\Delta_K}{\Delta}$$

$$\frac{C}{R} = \frac{(1\times 2\times 3\times 4\times 1)+(1\times 5\times 1)\times(1+3)}{1-(-2-3-4-5)+(-2)\times(-4)}$$

$$\frac{C}{R} = \frac{24+20}{1+14+8} = \frac{44}{23}$$

2. Compensators and Controllers

2.1 (d)

$$\theta = \tan^{-1}\left(\frac{\omega}{z_1}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right)$$

For phase lead controller

$$\theta > 0$$

$$\tan^{-1}\left(\frac{\omega}{z_1}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right) > 0$$

$$\tan^{-1}\left(\frac{\omega}{z_1}\right) > \tan^{-1}\left(\frac{\omega}{p_1}\right)$$

$$p_1 > z_1$$

2.2 (b, c)

The integral mode improves steady state performance and the derivative mode improves the transient performance.

2.3 (True)

The tachometer feedback is a derivative feedback.

So, tachometer adds zero at origin.

Hence, type decreases and stability is improved **(True)**.

3. Time Response Analysis

3.1 (a)

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0}\frac{K}{s(s+10)} = \infty$$

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+\infty} = 0$$

3.2 (b)

The steady state error of a stable 'type 0' unity feedback system for a unit step function is $\frac{1}{1+K_p}$.

3.3 (a)

$$\omega_n^2 = 25$$
$$\omega_n = 5$$
$$2\xi\omega_n = 8$$
$$\xi = \frac{8}{2\omega_n} = \frac{4}{5} = 0.8$$
$$\omega_d = \omega_n\sqrt{1-\xi^2} = 5\sqrt{1-(0.8)^2} = 3$$

For 2nd peak $n = 3$

$$t_p = \frac{n\pi}{\omega_d} = \frac{3\pi}{3} = \pi \text{ sec.}$$

3.4 (a)

$$K_V = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0}\frac{s\,4(1+2s)}{s^2(s+2)}$$
$$= \lim_{s\to 0}\frac{4(1+2s)}{s(s+2)} = \infty$$
$$e_{ss} = \frac{A}{K_V} = \frac{A}{\infty} = 0$$

3.5 Sol.

The poles of a continuous time oscillator are pure imaginary.

3.6 Sol.

a - 1, b - 5, c - 2.

(a) Overdamped → (1) poles on the negative real axis $\xi > 1$.

$$s = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

(b) Critically damped → (5) Multiple poles on the negative real axis.

$$\xi = 1$$
$$s = -\omega_n, -\omega_n$$

(c) Oscillatory → (2) Poles on the imaginary axis.

$$\xi = 0$$
$$s = \pm j\omega_n$$

3.7 Sol.

(a) - 1, (b) - 5, (c) - 4.

(a) Very low response at very high frequencies → Low pass system.

(b) Overshoot → Damping ratio.

(c) Synchro-control transformer output → Phase-sensitive modulation.

4. Stability Analysis

4.1 (d)

Using R-H criterion:

$$\begin{array}{c|ccc}
s^4 & 1 & 5 & K+10 \\
s^3 & 3 & 6 & 0 \\
s^2 & 3 & K+10 & 0 \\
s^1 & \frac{-12-3K}{3} & 0 & 0 \\
s^0 & K+10 & &
\end{array}$$

For stable system, all the coefficients of 1st column should be positive.

So, $\frac{-12-3K}{3} > 0$

$$-12 - 3K > 0$$
$$-12 > +3K$$
$$-4 > K$$

and $K + 10 > 0$

$$K > -10$$

So, the range of K

$$-10 < K < -4$$

4.2 (d)

Characteristic equation

$1 + GH = 0$

$$1+\frac{(1+sT_i)}{s}\times\frac{1}{s(1+sT)}\times 1 = 0$$

$s^2(1 + sT) + (1 + sT_i) = 0$

$s^3T + s^2 + sT_i + 1 = 0$

Using R-H method

$$\begin{array}{c|cc}
s^3 & T & T_i \\
s^2 & 1 & 1 \\
s^1 & T_i - T & \\
s^0 & 1 &
\end{array}$$

For stability, 1st column should be positive.

So, $T_i - T > 0$

$$T_i > T$$

4.3 (b)

$s^3 + 6Ks^2 + (K+2)\,s + 8 = 0$

Using R-H criteria

$$\begin{array}{c|cc} s^3 & 1 & K+2 \\ s^2 & 6K & 8 \\ s^1 & \dfrac{6K^2+12K-8}{6K} & 0 \\ s^0 & 8 & \end{array}$$

For stable system, 1st column element should be positive.

$$\frac{6K^2+12K-8}{6K} > 0$$

$6K^2 + 12K - 8 > 0$

$3K^2 + 6K - 4 > 0$

$$K = \frac{-6 \pm \sqrt{36+48}}{6} = -2.528,\ +0.528$$

$K > 0.528$

$K > -2.528$

So, $K > 0.528$

So, from given option for

$K = 2$ system will be stable.

4.4 (c)

$s^3 + 3s^2 + 4s + A = 0$

Using R-H criteria

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 3 & A \\ s^1 & \dfrac{12-A}{3} & 0 \\ s^0 & A & \end{array}$$

For stable system

$$A > 0$$

and $\dfrac{12-A}{3} > 0$

$12 - A > 0$

$12 > A$

$0 < A < 12$

4.5 (False)

$$\text{T.F. of } G(s) = \frac{(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)}$$

$$\text{T.F. of } F(s) = \frac{(s+P_1)(s+P_2)}{(s+Z_1)(s+Z_2)}$$

The condition for stability is that none of the pole of $G(s)$ should be on the right half of s-plane, but $G(s)$ may have zeros in the right half of s-plane. These zeros become pole of $F(s)$. Therefore the $F(s)$ need not be stable **(False)**.

5. Root Locus

5.1 (d)

$$G(s) = \frac{1}{(s+0.1)(s+2)}$$

$$\text{T.F.} = \frac{K\cdot G(s)}{1+KG(s)}$$

$$\text{T.F.} = \frac{\dfrac{K}{(s+0.1)(s+2)}}{1+\dfrac{K}{(s+0.1)(s+2)}}$$

$$\text{T.F.} = \frac{K}{(s+0.1)(s+2)+K}$$

$$\text{T.F.} = \frac{K}{K+0.2(1+10s)(1+0.5s)}$$

5.2 (a)

(b) & (d) options are wrong, because root locus is symmetrical about real axis.

Option (c) is wrong because root locus directions are from pole to zeros.

5.4 (b)

$$\text{O.L.T.F} = G(s)\,H(s) = \frac{K(s+1)}{s^3+5s^2+6s}$$

$$= \frac{K(s+1)}{s(s^2+5s+6)}$$

$$G(s)H(s) = \frac{K(s+1)}{s(s+2)(s+3)}$$

Poles → $0, -2, -3$

Zeros → -1

$$\text{Centroid} = \frac{(-2-3)-(-1)}{3-1}$$

$$= \frac{-5+1}{2} = -2 = (-2, 0)$$

5.5 (a)

$P = 3$

$z = 0$

$P - z = 3 - 0 = 3$

$$\theta = \frac{(2q+1)\,180^\circ}{P-z}$$

$\theta = 60^\circ, 180^\circ, 300^\circ$

So, only option (a) satisfy this.

5.6 (d)

$(n - m)$

difference between poles and zeros gives number of asymptotes.

6. Frequency Response Analysis

6.1 (b)

$$\begin{aligned} P &= 14 \\ z &= 2 \\ P - z &= 14 - 2 = 12 \\ \text{Slope} &= -20(P - z) = -20(12) \\ &= -240 \text{ dB/decade} \end{aligned}$$

6.2 (c)

$$\begin{aligned} G(s) &= \frac{10}{s(s+1)^2} \\ \angle G &= -180° = -90° - 2\tan^{-1}(\omega) \\ 2\tan^{-1}(\omega) &= 45° \\ \omega &= 1 \\ \omega_{pc} &= 1 \text{ rad/sec.} \\ |G|_{\omega = \omega_{pc}} &= \frac{10}{\omega(1+\omega^2)} = \frac{10}{1(1+1)} = 5 \end{aligned}$$

At $\omega = \omega_{pc}$ the polar plot crosses the negative real axis at –5.

6.3 (c)

$\omega_{pc} \Rightarrow$

$$\begin{aligned} -3\tan^{-1}(\omega_{pc}) &= -180° \\ \tan^{-1}(\omega_{pc}) &= 60° \\ \omega_{pc} &= \sqrt{3} \text{ rad/sec.} \\ |\text{G.H.}| &= X = \frac{1}{\left(\sqrt{1+\omega_{pc}^2}\right)} \\ &= \frac{1}{\left(\sqrt{1+(\sqrt{3})^2}\right)^3} = \frac{1}{8} \\ \text{G.M.} &= \frac{1}{X} = 8 \end{aligned}$$

6.4 Sol.

Corner frequency = 1 rad/sec.

$$\begin{aligned} \frac{1}{T} &= 1 \\ T &= 1 \\ y &= mX + C \\ 20 &= 0 + C \\ C &= 20 \\ C &= 20\log(k) = 20 \\ \log(k) &= 1 \\ k &= 10 \\ \text{T.F.} &= \frac{k}{(1+Ts)} = \frac{10}{(1+s)} \end{aligned}$$

6.5 (c)

$$\omega_n\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}$$

6.6 (–10)

At 180° $\rightarrow |GH| = 1.2$

$$\begin{aligned} \text{G.M.} &= 20\log\left(\frac{1}{|GH|}\right) = 20\log\left(\frac{1}{1.2}\right) \\ &= -1.6 \text{ dB} \end{aligned}$$

At $|GH| = 1 \quad \rightarrow \phi = -190°$

$$\begin{aligned} \text{P.M.} &= 180° + \phi \\ &= 180° - 190° = -10° \end{aligned}$$

Since both G.M. and P.M. are negative. So, the system is unstable.

7. State Space Analysis

7.1 Sol.

$$A\begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s+2 & 0 \\ 0 & s+4 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+2)(s+4)}\begin{bmatrix} s+4 & 0 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+4} \end{bmatrix}$$

State transition matrix

$$\phi(t) = e^{At} = L^{-1}\{[sI - A]^{-1}\}$$

$$\phi(t) = e^{At} = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-4t} \end{bmatrix}$$

7.2 (b)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ +2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

$$Q_c = [B \;\; AB]$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -2 & 4 \\ +2 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 4 \\ 1 & -1 \end{bmatrix}$$

$$|Q_c| = 0 - 4 = -4 \neq 0$$

Hence the given system is controllable.

Characteristic equation

$$[sI - A] = 0$$

$$\begin{bmatrix} s+2 & -4 \\ -2 & s+1 \end{bmatrix} = 0$$

$$(s + 2)(s + 1) - 8 = 0$$

$$s^2 + 3s - 6 = 0$$

$$s = \frac{-3 \pm \sqrt{9+24}}{2}$$

$$s = -4.37, 1.37$$

Since 1 pole lie in R.H.S. of s-plane.

So, the given system is unstable.

7.3 (b) & (c)

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$Q_c = [B \;\; AB]$$

$$Q_c = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix}$$

$$|Q_c| = 0 - 0 = 0$$

So, the given system is not completely controllable.

$$A^T = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$C = [1 \;\; 2]$$

$$C^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

$$Q_o = [C^T \;\; A^T C^T]$$

$$Q_o = \begin{bmatrix} 1 & -1 \\ 2 & -4 \end{bmatrix}$$

$$|Q_o| = -4 - (-2) = -2 \neq 0$$

So, the given system is completely observable.

(b) The system is not completely controllable.

(c) The system is completely observable.

■■■

Electronic Devices and Circuits

UNIT IV

CONTENTS

IV

Electronic Devices and Circuits

1. Basic Semiconductor Physics

1.1 Consider two energy levels: E_1, E eV above the Fermi level and E_2, E eV below the Fermi level. P_1 and P_2 are respectively the probabilities of E_1 being occupied by an electron and E_2 being empty. Then
(a) $P_1 > P_2$
(b) $P_1 = P_2$
(c) $P_1 < P_2$
(d) P_1 and P_2 depend on number of free electrons
[1987 : 2 Marks]

1.2 In an intrinsic semiconductor the free electron concentration depends on
(a) effective mass of electrons only
(b) effective mass of holes only
(c) temperature of the semiconductor
(d) width of the forbidden energy band of the semiconductor
[1987 : 2 Marks]

1.3 According to the Einstein relation, for any semiconductor the ratio of diffusion constant to mobility of carriers
(a) depends upon the temperature of the semiconductor
(b) depends upon the type of the semi conductor
(c) varies with life time of the semi conductor
(d) is a universal constant
[1987 : 2 Marks]

1.4 Direct band gap semi conductors
(a) exhibit short carrier life time and they are used for fabricating BJT's
(b) exhibit long carrier life time and they are used for fabricating BJT's
(c) exhibit short carrier life time and they are used for fabricating lasers
(d) exhibit long carrier life time and they are used for fabricating lasers
[1987 : 2 Marks]

1.5 Due to illumination by light, the electron and hole concentrations in a heavily doped N type semi conductor increases by Δn and Δp respectively if n_i is the intrinsic concentration then,
(a) $\Delta n < \Delta p$ (b) $\Delta n > \Delta p$
(c) $\Delta n = \Delta p$ (d) $\Delta n \times \Delta p = n_i^2$
[1989 : 2 Marks]

1.6 The concentration of ionized acceptors and donors in a semi conductor are N_A, N_D respectively. If $N_A > N_D$ and n_i is the intrinsic concentration, the position of the fermi level with respect to the intrinsic level depends on
(a) $N_A - N_D$ (b) $N_A + N_D$
(c) $\frac{N_A N_D}{n_i^2}$ (d) n_i
[1989 : 2 Marks]

1.7 Under high electric fields, in a semiconductor with increasing electric field,
(a) the mobility of charge carriers decreases
(b) the mobility of the carries increases
(c) the velocity of the charge carriers saturates
(d) the velocity of the charge carriers increases
[1990 : 2 Marks]

1.8 A silicon sample is uniformly doped with 10^{16} phosphorus atoms/cm^3 and 2×10^{16} boron atoms/cm^3. If all the dopants are fully ionized, the material is
(a) n-type with carrier concentration of 10^{16}/cm^3
(b) p-type with carrier concentration of 10^{16}/cm^3
(c) p-type with carrier of 2×10^{16}/cm^3
(d) n-type with a carrier concentration of 2×10^{16}/cm^3
[1991 : 2 Marks]

1.9 A semi conductor is irradiated with light such that carriers are uniformly generated throughout its volume. The semiconductor is n-type with $N_D = 10^{19}$/cm^3. If the excess electron concentration in the steady state is $\Delta n = 10^{15}$/cm^3 and if $\tau_p = 10$ μsec. (minority carries life time) the generation rate due to irradiation
(a) is 10^{20} e-h pairs/cm^3/s
(b) is 10^{24} e-h pairs/cm^3/s
(c) is 10^{10} e-h pairs/cm^3/s
(d) cannot be determined, the given data is insufficient
[1992 : 2 Marks]

1.10 A p-type silicon sample has a higher conductivity compared to an n-type silicon sample having the same dopant concentration. (True/False)

[1994 : 1 Mark]

2. PN-Junction Diodes and Special Diodes

2.1 The diffusion capacitance of a p-n junction

(a) decreases with increasing current and increasing temperature

(b) decreases, with decreasing current and increasing temperature

(c) increases with increasing current and increasing temperature

(d) does not depend on current and temperature

[1987 : 2 Marks]

2.2 For a pn-junction match the type of breakdown with phenomenon

1. Avalanche breakdown
2. Zener breakdown
3. Punch through

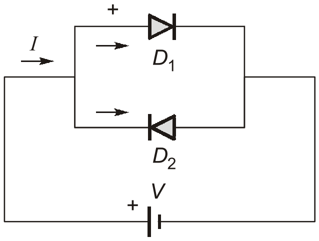
A. Collision of carriers with crystal ions
B. Early effect
C. Rupture of covalent bond due to strong electric field.

(a) 1-B, 2-A, 3-C (b) 1-C, 2-A, 3-B

(c) 1-A, 2-B, 3-C (d) 1-A, 2-C, 3-B

[1988 : 2 Marks]

2.3 In the circuit shown below the current voltage relationship when D_1 and D_2 are identical is given by (Assume Ge diodes)



(a) $V = \frac{kT}{q} \sinh\left(\frac{I}{2}\right)$

(b) $V = \frac{kT}{q} In\left(\frac{I}{I_0}\right)$

(c) $V = \frac{kT}{q} \sinh^{-1}\left(\frac{I}{2}\right)$

(d) $V = \frac{kT}{q} [Exp(-I) - 1]$

[1988 : 2 Marks]

2.4 The switching speed of P^+N junction (having a heavily doped P region) depends primarily on

(a) the mobility of minority carriers in the P^+-region.

(b) the lifetime of minority carriers in the P^+-region

(c) the mobility of majority carriers in the N-region

(d) the lifetime of majority carriers in the N-region

[1989 : 2 Marks]

2.5 In a Zener diode

(a) only the P-region is heavily doped

(b) only the N-region is heavily doped

(c) both P and N-regions are heavily doped

(d) both P and P-regions are lightly doped

[1989 : 2 Marks]

2.6 In a junction diode

(a) the depletion capacitance increases with increase in the reverse bias

(b) the depletion capacitance decreases with increase in the reverse bias

(c) the depletion capacitance increases with increase in the forward bias

(d) the depletion capacitance is much higher than the depletion capacitance when it is forward biased

[1990 : 1 Mark]

2.7 In a uniformly doped abrupt p-n junction the doping level of the n-side is four(4) times the doping level of the p-side the ratio of the depletion layer width of n-side verses p-side is

(a) 0.25 (b) 0.5

(c) 1.0 (d) 2.0

[1990 : 2 Marks]

2.8 The small signal capacitance of an abrupt P^+n junction is 1 nF/cm^2 at zero bias. If the built-in voltage is 1 volt, the capacitance at a reverse bias voltage of 99 volts is

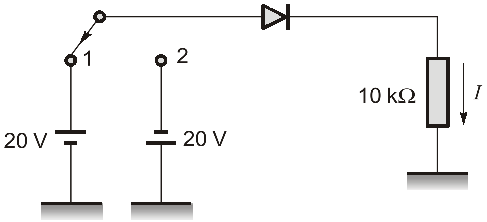
(a) 10 (b) 0.1

(c) 0.01 (d) 100

[1991 : 2 Marks]

2.9 Referring to the below figure the switch S is in position 1 initially and steady state condition exist from time $t = 0$ to $t = t_0$, the switch is suddenly thrown into position 2. The current I through the 10 K resistor as a function of time t, from $t = 0$ is?

(give the sketch showing the magnitudes of the current at $t = 0$, $t = t_0$ and $t = \infty$).



[1991 : 2 Marks]

2.10 The built-in potential (diffusion potential) in a *p-n* junction

(a) is equal to the difference in the Fermi-level of the two sides, expressed in volts
(b) increases with the increase in the doping levels of the two sides
(c) increases with the increase in temperature
(d) is equal to the average of the Fermi levels of the two sides

[1993 : 2 Marks]

3. BJT and FET Basics

3.1 The pinch off voltage for a *n*-channel JFET is 4 V, when $V_{GS} = 1$ V, the pinch-off occurs for V_{DS} equal to

(a) 3 V
(b) 5 V
(c) 4 V
(d) 1 V

[1987 : 2 Marks]

3.2 In an *n*-channel JFET, V_{GS} is held constant. V_{DS} is less than the breakdown voltage. As V_{DS} is increased

(a) conducting cross-sectional area of the channel '*S*' and the channel current density '*J*' both increase
(b) '*S*' decrease and '*J*' decrease
(c) '*S*' decreases and '*J*' increase
(d) '*S*' increases and '*J*' decreases

[1988 : 2 Marks]

3.3 In MOSFET devices the *n*-channel type is better than the *P*-channel type in the following respects

(a) it has better noise immunity
(b) it is faster
(c) it is TTL compatible
(d) it has better drive capability

[1988 : 2 Marks]

3.4 In a MOSFET, the polarity of the inversion layer is the same as that of the

(a) charge on the gate electrode
(b) minority carriers in the drain
(c) majority carriers in the substrate
(d) majority carriers in the source

[1989 : 2 Marks]

3.5 The 'Pinch-off' voltage of a JFET is 5.0 volts, Its 'cut-off' voltage is

(a) $(5.0)^{1/2}$ V
(b) 2.5 V
(c) 5.0 V
(d) $(5.0)^{3/2}$ V

[1990 : 2 Marks]

3.6 Which of the following effects can be caused by a rise in the temperature?

(a) increase in MOSFET current (I_{DS})
(b) increase in BJT current (I_C)
(c) decrease in MOSFET current (I_{DS})
(d) decrease in BJT current (I_C)

[1990 : 2 Marks]

3.7 In a transistor having finite β, forward bias across the base emitter junction is kept constant and the reverse bias across the collector base junction is increased. Neglecting the leakage across the collector base junction and the depletion region generations current, the base current will ____ (increase/decrease/remains constant).

[1992 : 2 Marks]

3.8 An n-channel JFET has a pinch-off voltage $V_p = -5$ V, V_{DS}(max) = 20 V, and $g_m = 2$ mA/V. The min 'ON' resistance is achieved in the JFET for

(a) $V_{GS} = -7$ V and $V_{DS} = 0$ V
(b) $V_{GS} = 0$ V and $V_{DS} = 0$ V
(c) $V_{GS} = 0$ V and $V_{DS} = 20$ V
(d) $V_{GS} = -7$ V and $V_{DS} = 20$ V

[1992 : 2 Marks]

3.9 The threshold voltage of an *n*-channel MOSFET can be increased by

(a) increasing the channel dopant concentration
(b) reducing the channel dopant concentration
(c) reducing the gate oxide thickness
(d) reducing the channel length

[1994 : 1 Mark]

3.10 The transit time of the current carries through the channel of a JFET decides its ______ characteristic

(a) source
(b) drain
(c) gate
(d) source and drain

[1994 : 1 Mark]

3.11 Channel current is reduced on application of a more positive voltage to the gate of the depletion mode *n*-channel MOSFET. (True/False)

[1994 : 1 Mark]

3.12 The break down voltage of a transistor with its base open is BV_{CEO} and that with emitter open is BV_{CBO}, then

(a) $BV_{CEO} = BV_{CBO}$
(b) $BV_{CEO} > BV_{CBO}$
(c) $BV_{CEO} < BV_{CBO}$
(d) BV_{CEO} is not related to BV_{CBO}

[1995 : 1 Mark]

3.13 Match the following:

List-I

A. The current gain of a BJT will be increased
B. The current gain of a BJT will be reduced
C. The break-down voltage of a BJT will be reduced

List-II

1. The collector doping concentration is increased
2. The base width is reduced
3. The emitter doping concentration to base doping concentration ratio is reduced
4. The base doping concentration is increased keeping the ratio of the emitter doping concentration to base doping concentration constant
5. The collector doping concentration is reduced

[1994 : 2 Marks]

3.14 The transit time of current carriers through the channel of an FET decides it ____ Characteristics.

[1994 : 1 Mark]

■■■

Answers Electronic Devices and Circuits

1.1	(b)	1.2	(c)	1.3	(a)	1.4	(c)	1.5	(c)	1.6	(a)	1.7	(a, c)
1.8	(b)	1.9	(a)	1.10	(False)	2.1	(b)	2.2	(d)	2.3	(b)	2.4	(d)
2.5	(c)	2.6	(b)	2.7	(a)	2.8	(b)	2.9	(sol.)	2.10	(a & b)	3.1	(a)
3.2	(c)	3.3	(b)	3.4	(d)	3.5	(c)	3.6	(b, c)	3.7	(sol.)	3.8	(b)
3.9	(b)	3.10	(b)	3.11	(False)	3.12	(c)	3.13	(A-2, B-3, C-1)			3.14	(sol.)

Explanations Electronic Devices and Circuits

1. Basic Semiconductor Physics

1.2 (c)

By mass action law:

$$\boxed{n \cdot p = n_i^2}$$

n_i = intrinsic carrier concentration
p = hole concentration
n = electron concentration

$$\boxed{n_i^2 \propto T^3}$$

$$n_i \propto T^{3/2}$$

For intrinsic semiconductor,

$$n = p = n_i$$
$$n \propto T^{3/2}$$

1.3 (a)

We know that

$$\boxed{\text{Einstein equation} \rightarrow \frac{D}{\mu} = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T = \frac{T(°K)}{11600}}$$

Where,

T = Temperature in °K
V_T = Thermal voltage
D = Diffusion constant
μ = Mobility
D_n = Electron diffusion constant
D_p = Hole diffusion constant
μ_n = Electron mobility
μ_p = Hole mobility

1.4 (c)

DBG (Direct Band Gap) semiconductors exhibit short carrier life time they are used for fabricating lasers. In DBG semiconductor during the recombination the energy is released in the form of light.

1.5 (c)

$$\Delta n = \Delta p$$

Due to illumination by light EHP (electron-hole pair) generation occurs.

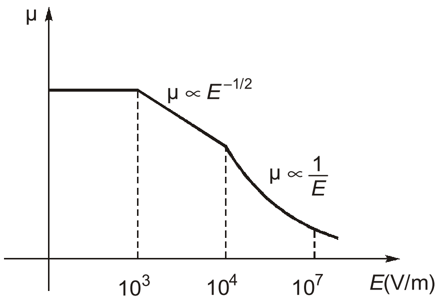
So, $\Delta n = \Delta p$

Where

Δn = increase in electron concentration due to illumination by light

Δp = increase in hole concentration due to illumination by light

1.7 (a) & (c)



$$\boxed{v_d = \mu E}$$

Where

v_d = Drift velocity

μ = Mobility

E = Applied electric field

For high electric field, with increasing electric field:

1. The mobility of charge carriers decreases as electric field increases.

$$\mu \propto \frac{1}{E}$$

2. The velocity (drift velocity) of charge carriers saturates.

1.8 (b)

P-type with carrier concentration of $10^{16}/\text{cm}^3$.

Given that,

$N_D = n$ = Phosphorus atoms $= 10^{16}/\text{cm}^3$

$N_A = p$ = Boron atoms $= 2 \times 10^{16}/\text{cm}^3$

$\because N_A >> N_D$

So, the resultant material will be p-type semiconductor carrier concentration

$$= N_A - N_D$$
$$= 2 \times 10^{16} - 10^{16}$$
$$= 10^{16}/\text{cm}^3$$

1.9 (a)

10^{20} e-h pairs/cm_3/s

Given that, $\Delta n = 10^{15}/\text{cm}^3$

$\tau_p = 10\ \mu\text{sec} = 10 \times 10^{-6}$ sec.

$$\text{Generation rate} = \frac{\Delta P}{\tau_p} = \frac{10^{15}}{10 \times 10^{-6}}$$

$= 10^{20}$ e-h pairs/cm^3/s

1.10 Sol. (False)

The given statement is false, because for a given semiconductor the electron mobility (μ_n) is always higher than the hole mobility (μ_p).

$\boxed{\mu_n > \mu_p}$ (for a given semiconductor) we know that, the conductivity of a given n-type semiconductor

$$\sigma_n = nq\mu_n$$

the conductivity of a given p-type semiconductor.

$$\sigma_p = pq\mu_p$$

given that $n = p$ = same dopant concentration

$$q = 1.602 \times 10^{-19} \text{ Col.}$$

So, $\boxed{\sigma_n > \sigma_p}$

2. PN-Junction Diodes and Special Diodes

2.1 (b)

Decreases with decreasing current and increasing temperature.

$$\text{Diffusion capacitance} = C_D = \tau g = \frac{\tau}{r}$$

$$r = \frac{\eta V_T}{I_f}$$

$$C_D = \frac{\tau I_f}{\eta V_T} = \frac{\tau I_f}{\eta kT}$$

$$C_D \propto I_f$$

$$C_D \propto \frac{1}{T}$$

2.2 (d)

1-A, 2-C, 3-B

Avalanche breakdown → Collision of carriers with crystal ions.

Zener breakdown → Rupture of covalent bond due to strong electric field.

Punch through → Early effect.

2.3 (b)

$$V = \frac{KT}{q} ln\left(\frac{I}{I_o}\right)$$

Diode D_1 is in forward bias

Diode D_2 is in reverse bias

So, the current through diode D_1 is forward current I_f and current through diode D_2 is reverse current I_o

So, total current = $I = I_f + I_o$

$$I = I_o\left(e^{V_d/\eta V_T} - 1\right) = I_o e^{V_d/\eta V_T} - I_o$$

$$I = \left(I_o e^{V_d/\eta V_T} - I_o\right) + I_o$$

$$I = I_o e^{V_d/\eta V_T}$$

$$e^{V_d/\eta V_T} = \frac{I}{I_o}$$

$$\frac{V_d}{\eta V_T} = ln\left(\frac{I}{I_o}\right)$$

$$V_d = \eta V_T ln\left(\frac{I}{I_o}\right)$$

$$V_d = \frac{KT}{q} ln\left(\frac{I}{I_o}\right) \qquad \{\text{For Ge, } \eta = 1\}$$

$$\therefore \quad V_d = V$$

$$V = \frac{KT}{q} ln\left(\frac{I}{I_o}\right)$$

2.4 (d)

The lifetime of majority carriers in the *N*-region.

The switching speed of a P^+N (heavily doped *p*-region) junction depends on the lifetime (τ) of majority carriers (electrons) in the *N*-region (lightly doped region).

2.5 (c)

Both *P* and *N*-regions are heavily doped.

In a Zener diode *P* and *N* both the regions are heavily doped.

Doping level of Zener diode is 1 : 10^5.

2.6 (b)

The depletion capacitance decreases with increase in the reverse bias.

Depletion width = *W*

$$W \propto \sqrt{V_{RB}}$$

$$W \propto \sqrt{\text{Reverse bias voltage}}$$

$$\text{Capacitance} = C = \frac{A\epsilon}{W}$$

$$C \propto \frac{1}{W}$$

$$C \propto \frac{1}{\sqrt{\text{Reverse bias voltage}}}$$

2.7 (a)

In the step graded diode, by using charge density condition or charge neutrality condition

$$\frac{W_N}{W_P} = \frac{N_A}{N_D}$$

$$\frac{W_N}{W_P} = \frac{N_A}{4N_A} = \frac{1}{4} = 0.25$$

2.8 (b)

For abrupt p-n junction

$$C_j \propto V^{-1/2}$$

$$C_j \propto \frac{1}{\sqrt{V}}$$

$$\frac{C_2}{C_1} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{1+0}{1+99}} = \sqrt{\frac{1}{100}} = \frac{1}{10}$$

$$C_2 = \frac{C_1}{10} = \frac{1}{10} = 0.1\,\text{nF/cm}^2$$

2.9 Sol.

When diode instantaneously switched from a conduction state it needs some time to return to non-conduction state, so diode behaves as short circuit for the little time, even in reverse direction. This is due to accumulation of stored excess minority carrier charge when diode is forward biased.

Time required to return back to state of non conduction is 'Reverse recovery time' which is 'storage time' and 'transition time'.

- Storage Time is the period for which diode remains in conduction state even in reverse direction.
- Transition time is time elapsed in returning back to state of non conduction.

For $0 < t < t_o$; $V_m = 20$ V, '*D*' is Forward biased

$$i = \frac{V_m}{R} = \frac{20}{10\,\text{k}\Omega} = 2\,\text{mA}$$

For $t > t_o$; $V_m = -20$

During storage time: $t_o < t < t_o + t_s$;

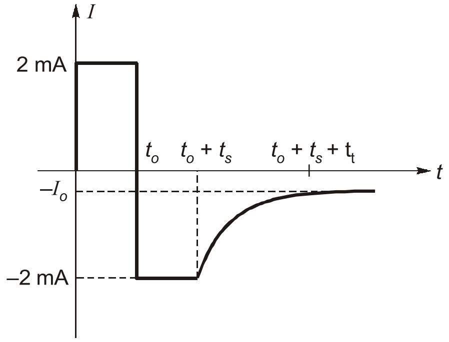
$$i = \frac{V_m}{R} = -\frac{20}{10\,\text{k}\Omega} = -2\,\text{mA}$$

During transition time: $t_o + t_s < t < t_o + t_s + t_t$

i decreases exponentially to $(-I_o)$

For $t > t_o + t_s + t_t$

$$i = (-I_o)$$



2.10 (a) & (b)

Increases with the increase in the doping levels of the two sides.

Built in potential or diffusion potential across a p-n junction diode.

$$V_o = kT\,ln\left(\frac{N_A N_D}{n_i^2}\right)$$

So, $\quad V_o \propto n_i^2$

$$V_o \propto N_A \cdot N_D$$

So, option (a) and option (b) both are correct.

3. BJT and FET Basics

3.1 (a)

Given that,

$$V_p = 4\text{ V}$$

$$V_{GS} = 1\text{ V}$$

$$|V_{DS}| = |V_p| - |V_{GS}|$$

$$|V_{DS}| = 4 - 1 = 3\text{ V}$$

3.2 (c)

V_{GS} is held constant and V_{DS} is increased then the depletion width increases, so the cross-sectional area of the channel 'S'.

$\because$ Current density $= \dfrac{I}{A} = \dfrac{I}{'S'} = J$

$$S\downarrow \rightarrow J\uparrow$$

So, the channel current density increases.

3.3 (b)

n-channel MOSFET is faster than the p-channel MOSFET.

Mobility of electrons is always higher than the mobility of holes.

$$\mu_n > \mu_p$$

In n-channel the charge carriers are electrons whereas in p-channel MOSFET the charge carriers are holes.

3.4 (d)

In a MOSFET, the polarity of the inversion layer is the same as that of the majority carriers in the source.

3.5 (c)

Pinch-off voltage = Cut-off voltage

So, cut-off voltage = 5.0 V

3.6 (b) & (c)

$$I_C = \beta I_b + (1+\beta) I_{CO}$$

as temperature increases, I_{CO} increases, so the current (I_C) increases in BJT with rise in temperature.

$\because$ mobility decreases as temperature increases.

$$T\uparrow \rightarrow \mu\downarrow$$

So, in MOSFET, current (I_{DS}) decreases with rise in temperature.

$$T\uparrow \rightarrow I_{DS}\downarrow$$

3.7 Sol. (decrease)

As the reverse bias increase at CB (collector base) junction then the collector current (I_c) increases and the effective base width decreases, so the recombination in base decreases.

3.8 (b)

$V_{GS} = 0$ V and $V_{DS} = 0$ V

For n-channel JFET, JFET offers minimum 'ON' resistance when V_{GS} is positive and large and voltage V_{DS} is very small, ideally $V_{DS} = 0$ V.

3.9 (b)

For the NMOS threshold voltage is given by

$$V_T = V_{To} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\,\right]$$

$$\gamma = \frac{\sqrt{2qN_A \epsilon_s}}{C_{ox}} = \frac{t_{ox}\sqrt{2qN_A \epsilon_s}}{3.45\times 10^{-11}}$$

So, the threshold voltage of an *n*-channel MOSFET can be increased by reducing the channel dopant concentration.

3.10 (b)

The transit time of the current carries through the channel of a JFET decides its drain characteristic.

3.11 Sol. (False)

For depletion mode n-channel MOSFET, if the GATE terminal is made more positive then the channel becomes more and more n-type hence the drain current will increase.

3.12 (c)

The relationship between open base breakdown voltage (BV_{CEO}) of BJT with open emitter voltage (BV_{CBO}) is given by

$$BV_{CEO} = BV_{CBO}\sqrt{\frac{1}{\beta}}$$

$\because \quad \beta > 1$

So, $BV_{CEO} < BV_{CBO}$

3.13 Sol.

(A-2, B-3, C-1)

- As the base width of the BJT is reduced then the recombination current (base current I_B) decreases as a result collector current (I_C) increases. So, the current gain of the BJT increases.

$$\boxed{\alpha = \frac{I_C}{I_E}}$$

- If the emitter doping concentration to base doping concentration ratio is reduced then the emitter injection efficiency decreases, so the current gain (α) of BJT reduces.
- If the collector doping concentration is increased then the breakdown (V_{BR}) of a BJT will be reduced.

3.14 Sol.

The transit time of a current carriers through the channel of an FET decides its ***switching*** characteristics.

■■■

Analog Circuits

UNIT V

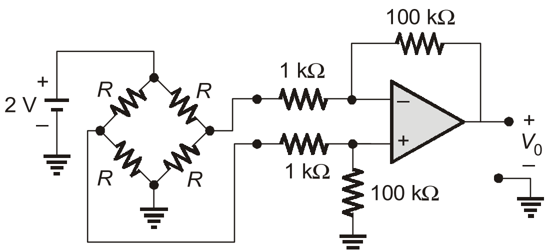
CONTENTS

V

Analog Circuits

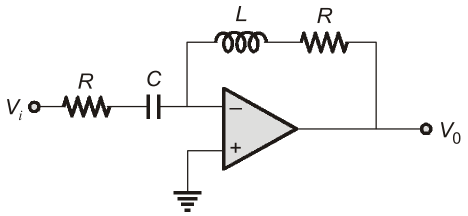
1. Operational Amplifiers

1.1 In figure shown below, if the CMRR of the operational amplifier is 60 dB, then the magnitude of the output voltage is:



[1987 : 2 Marks]

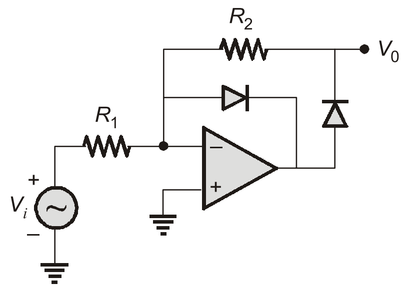
1.2 The Op-Amp shown in figure below is ideal. $R = \sqrt{L/C}$. The phase angle between V_0 and V_i, at $\omega = 1/\sqrt{LC}$



(a) $\pi/2$ (b) π
(c) $3\pi/2$ (d) 2π

[1988 : 2 Marks]

1.3 Refer to figure shown below:



(a) For $V_i > 0$, $V_0 = -\frac{R_2}{R_1}V_i$

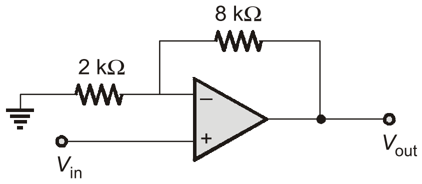
(b) For $V_i > 0$, $V_0 = 0$

(c) For $V_i < 0$, $V_0 = -\frac{R_2}{R_1}V_i$

(d) For $V_i < 0$, $V_0 = 0$

[1989 : 2 Marks]

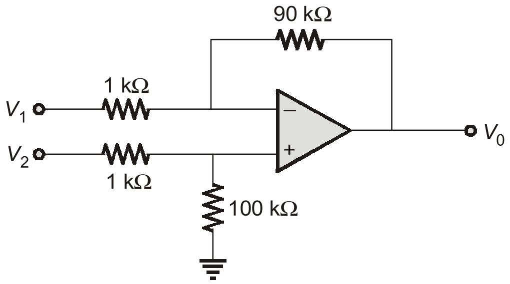
1.4 The Op-Amp of figure shown below has a very poor open loop voltage gain of 45 but is otherwise ideal. The gain of the Amplifier equals:



(a) 5 (b) 20
(c) 4 (d) 4.5

[1990 : 2 Marks]

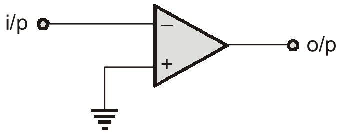
1.5 The CMRR of the differential Amplifier of the figure shown below is equal to



(a) ∞ (b) 0
(c) 1000 (d) 1800

[1990 : 2 Marks]

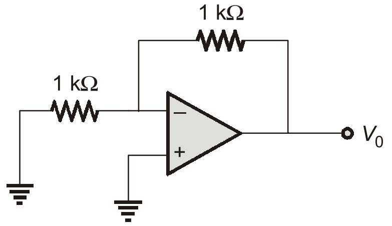
1.6 If the input to the circuit of figure is a sine wave the output will be



(a) A half-wave rectified sine wave
(b) A full-wave rectified sine wave
(c) A triangular wave
(d) A square wave

[1990 : 2 Marks]

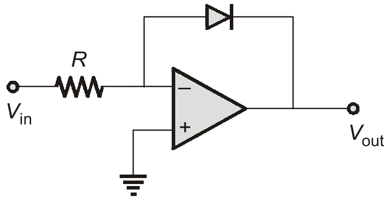
1.7 An Op-Amp has an offset voltage of 1 mV and is ideal in all other respects. If this Op-Amp is used in the circuit shown in fig. The output voltage will be (Select the nearest value).



(a) 1 mV
(b) 1 V
(c) ± 1 V
(d) 0 V

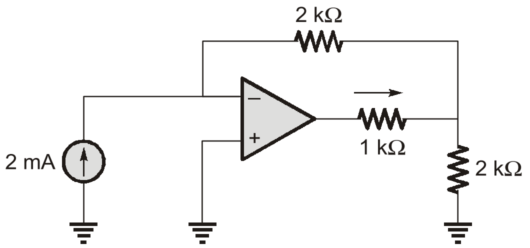
[1992 : 2 Marks]

1.8 The circuit of fig. uses on ideal OP-Amp for small positive values of V_{in}, the circuit works as



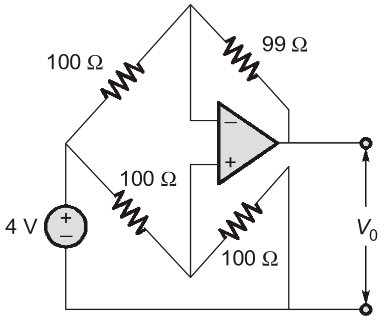
(a) a half wave rectifier
(b) a differentiator
(c) a logarithmic Amplifier
(d) an exponential Amplifier [1992 : 2 Marks]

1.9 Assume that the operational amplifier in figure is ideal the current I through the 1 kΩ resistor is __.



[1992 : 2 Marks]

1.10 For the ideal Op-Amp circuit of fig. Determine the output voltage V_0



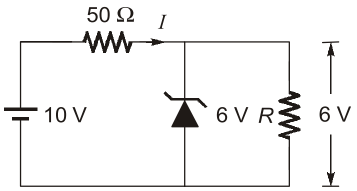
[1993 : 2 Marks]

1.11 The frequency compensation is used in OP-Amps to increase its ______.

[1994 : 1 Mark]

2. Diodes Applications

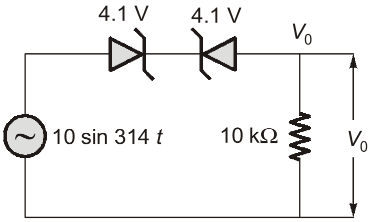
2.1 The 6 V Zener diode shown below has zero Zener resistance and a knee current of 5 mA. The minimum value of R. So that the voltage across it does not fall below 6 V is



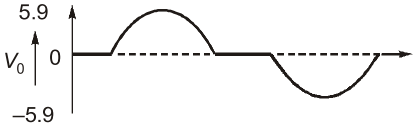
(a) 1.2 kΩ
(b) 50 Ω
(c) 80 Ω
(d) 0 Ω

[1992 : 2 Marks]

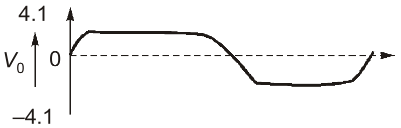
2.2 The wave shape of V_0 in figure is



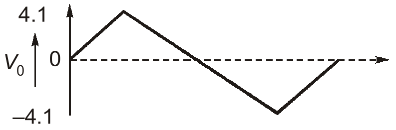
(a)



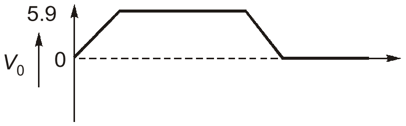
(b)



(c)



(d)



[1993 : 1 Mark]

3. BJT Analysis

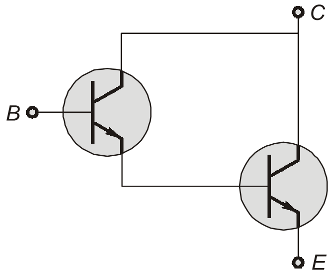
3.1 The configuration of cascode amplifier is
(a) *CE* – *CE* (b) *CE* – *CB*
(c) *CC* – *CB* (d) *CC* – *CC*

[1987 : 2 Marks]

3.2 The quiescent collector current I_C, of a transistor is increased by changing resistances. As a result
(a) g_m will not be affected
(b) g_m will decrease
(c) g_m will increase
(d) g_m will increase or decrease depending upon bias stability.

[1988 : 2 Marks]

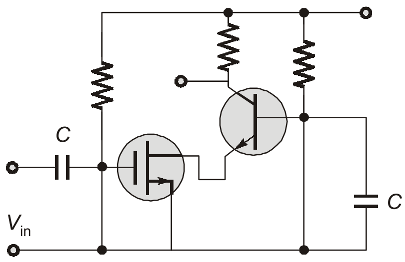
3.3 Each transistor in the Darlington pair (see Fig. below) has h_{FE} = 100. The overall h_{FE} of the composite transistor neglecting the leakage currents is



(a) 10000 (b) 10001
(c) 10100 (d) 10200

[1988 : 2 Marks]

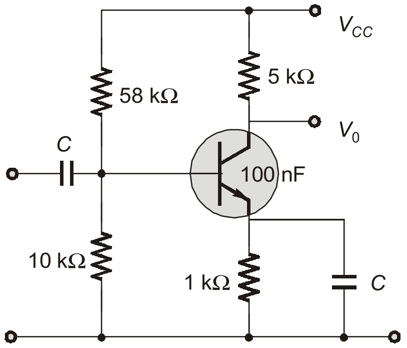
3.4 The amplifier circuit shown below uses a composite transistor of a MOSFET and BIPOLAR in cascade. All capacitances are large. g_m of the MOSFET = 2 mA/V, and h_{fe} of the BIPOLAR = 99. The overall transconductance g_m of the composite transistor is



(a) 198 mA/V (b) 9.9 mA/V
(c) 4.95 mA/V (d) 1.98 mA/V

[1988 : 2 Marks]

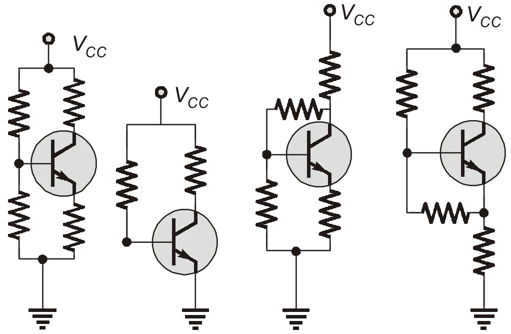
3.5 The transistor in the amplifier shown below has following parameters :
h_{fe} = 100, h_{ie} = 2 kΩ, h_{re} = 0, h_{oe} = 0.05 mhos. *C* is very large. The output impedance is



(a) 20 kΩ (b) 16 kΩ
(c) 5 kΩ (d) 4 kΩ

[1988 : 2 Marks]

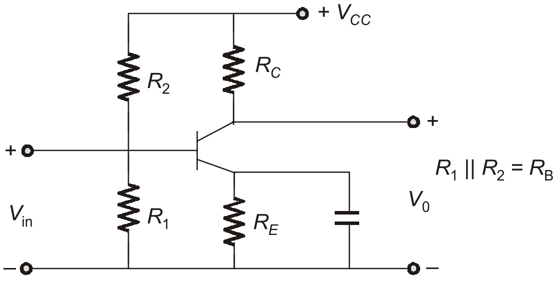
3.6 Of the four biasing circuits shown in Fig. For a BJT, indicate the one which can have maximum bias stability



(a) (b) (c) (d)

[1989 : 2 Marks]

3.7 For good stabilized biasing of the transistor of the *CE* Amplifier of fig, we should have



(a) $\frac{R_E}{R_B} << 1$ (b) $\frac{R_E}{R_B} >> 1$

(c) $\frac{R_E}{R_B} << h_{FE}$ (d) $\frac{R_E}{R_B} >> h_{FE}$

[1990 : 2 Marks]

3.8 Which of the following statements are correct for basic transistor Amplifier configurations?

(a) CB Amplifiers has low input impedance and low current gain

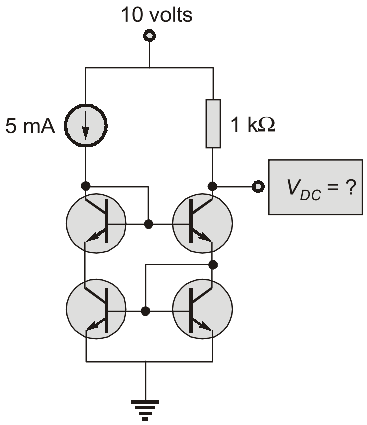
(b) CC Amplifiers has low output impedance and a high current gain.

(c) CE Amplifier has very poor voltage gain but very high input impedance

(d) The current gain of CB Amplifier is higher than the current gain of CC Amplifiers

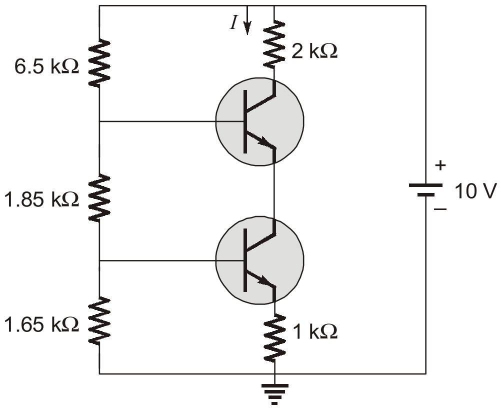
[1990 : 2 Marks]

3.9 In figure all transistors are identical and have a high value of beta. The voltage V_{DC} is equal to ____.



[1991 : 2 Marks]

3.10 If the transistor in fig. has high value of β and V_{BE} of 0.65 the current I flowing through the 2 kilo ohms resistance will be __________.



[1992 : 2 Marks]

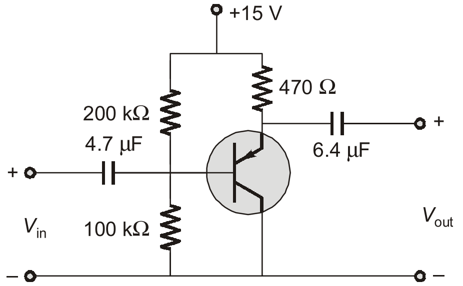
3.11 The Bandwidth of an n-stage tuned Amplifier, with each stage having a bandwidth of B, is given by

(a) B/n (b) $B/\sqrt{n}$

(c) $B\sqrt{2^{1/n}-1}$ (d) $B/\sqrt{2^{1/n}-1}$

[1993 : 1 Mark]

3.12 For the Amplifier circuit of fig. The transistor has a β of 800. The mid band voltage gain V_0/V_i of the circuit will be.



(a) 0 (b) < 1

(c) ≈ 1 (d) 800

[1993 : 1 Mark]

3.13 α cut-off frequency of a bipolar junction transistor

(a) increase with the increase in base width

(b) increase with the increase in emitter width

(c) increases with the increase in collector width

(d) increase with decrease in the base width

[1993 : 2 Marks]

3.14 In order to reduce the harmonic distortion in an Amplifier its dynamic range has to be ____.

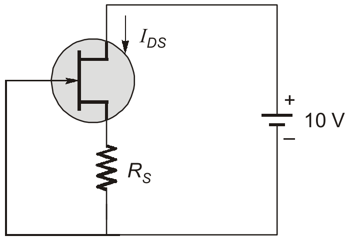
[1994 : 1 Mark]

3.15 A Common Emitter transistor Amplifier has a collector current of 1.0 mA when its a base current is 25 µA at the room temperature. It's input resistance is approximately equal to ________.

[1994 : 1 Mark]

4. FET and MOSFET Analysis

4.1 The JFET in the circuit shown in fig. has an I_{DSS} = 10 mA and V_P = – 5 V. The value of the resistance R_S for a drain current I_{DS} = 6.4 mA is (Select the Nearest value)



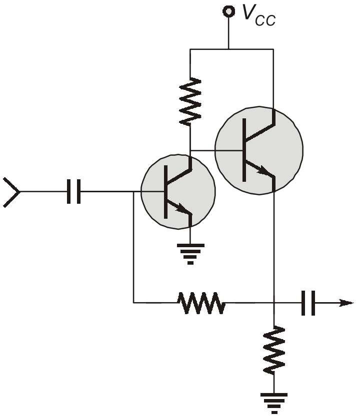
(a) 150 ohms (b) 470 ohms
(c) 560 ohms (d) 1 Kilo ohm
[1992 : 2 Marks]

5. Frequency Response of Amplifier

5.1 In a multi-stage RC-Coupled Amplifier the coupling capacitor.
(a) Limits the low frequency response
(b) Limits the high frequency response
(c) Does not effect the frequency response
(d) Blocks the d.c components without effecting the frequency response.
[1993 : 1 Mark]

6. Feedback Amplifiers

6.1 The feedback amplifier shown in Fig. has:



(a) current – series feedback with large input impedance and large output impedance.
(b) voltage – series feedback with large input impedance and low output impedance.
(c) voltage – shunt feedback with low input impedance and low output impedance.
(d) current – shunt feedback with low input impedance and output impedance.
[1989 : 2 Marks]

6.2 Two non-inverting amplifiers, one having a unity gain and the other having a gain of twenty, are made using identical operational amplifiers. As compared to the unity gain amplifier, the amplifier with gain twenty has
(a) less negative feedback
(b) greater input impedance
(c) less bandwidth
(d) none of the above. [1991 : 2 Marks]

6.3 Negative feed back in Amplifiers
(a) improves the signal to noise ratio at the input
(b) improves the signal to noise ratio at the output
(c) does not effect the signal to noise ratio at the output
(d) reduces distortion [1993 : 1 Mark]

7. Oscillator Circuits

7.1 Match the following

List-I	List-II
(a) Hartley	(1) Low frequency oscillator
(b) Wein-bridge	(2) High frequency oscillator
(c) Crystal	(3) Stable frequency oscillator
	(4) Relaxation oscillator
	(5) Negative Resistance oscillator

[1994 : 2 Marks]

8. Power Amplifiers

8.1 In case of class *A* amplifiers the ratio (efficiency of transformer coupled amplifier)/(efficiency of a transformer less amplifier) is
(a) 2.9 (b) 1.36
(c) 1.0 (d) 0.5
[1987 : 2 Marks]

8.2 In a transistor push-pull Amplifier
(a) there is no d.c present in the output
(b) there is no distortion in the output
(c) there is no even harmonics in the output
(d) there is no add harmonics in the output
[1993 : 1 Mark]

8.3 A Class – *A* transformer coupled, transistor power Amplifier is required to deliver a power output of 10 watts. The maximum power Rating of the transistor should not be less than
(a) 5 W (b) 10 W
(c) 20 W (d) 40 W
[1994 : 1 Mark]

■■■

Answers Analog Circuits

1.1	(100)	1.2	(c)	1.3	(b, c)	1.4	(d)	1.5	(c)	1.6	(d)	1.7	(*)
1.8	(c)	1.9	(–4)	1.10	(0.02)	1.11	(sol.)	2.1	(c)	2.2	(a)	3.1	(b)
3.2	(c)	3.3	(d)	3.4	(d)	3.5	(d)	3.6	(a)	3.7	(b)	3.8	(a, b)
3.9	(5)	3.10	(1)	3.11	(c)	3.12	(c)	3.13	(d)	3.14	(large)	4.1	(a)
5.1	(a)	6.1	(c)	6.2	(a, c)	6.3	(b, d)	7.1	(a-2, b-1, c-3)			8.1	(b)
8.2	(a, c)	8.3	(c)										

Explanations Analog Circuits

1. Operational Amplifiers

1.1 Sol.

By voltage divider rule:

$$V^- = 2\frac{R}{R+R} = 1\text{ V} \quad ; \quad V^+ = 2\frac{R}{R+R} = 1\text{ V}$$

So, $\boxed{v_d = V^+ - V^-} = 1 - 1 = 0\text{ V}$

$$\boxed{V_c = \frac{V^+ + V^-}{2}} = \frac{1+1}{2} = 1\text{ V}$$

$$A^- = -\frac{R_f}{R_1} = -\frac{100\,k}{1\,k} = -100$$

$$A^+ = \left(1+\frac{R_f}{R_1}\right)\times\left(\frac{100}{100+1}\right) = \left(1+\frac{100\,k}{1}\right)\left(\frac{100}{101}\right)$$

$$= 100$$

$$\boxed{A_d = \frac{A^+ - A^-}{2}}$$

$$A_d = \frac{100-(-100)}{2} = 100$$

$\because$ CMRR = 60 dB = 10^3

So, $$\frac{A_d}{A_c} = 10^3$$

$\Rightarrow$ $$A_c = \frac{A_d}{10^3} = \frac{1}{10}$$

Now $$V_o = |A_c|\,V_c + |A_d|\,v_d$$

$\therefore$ $$V_o = |A_c|\,V_c = \frac{1}{10}\times 1 = 100\text{ mV}$$

1.2 (c)

$\boxed{\dfrac{V_o}{V_i} = -\dfrac{Z_f}{Z_1}}$ ...For inverting amplifiers

$$\frac{V_o}{V_i} = -\frac{(R+j\omega L)}{R+\dfrac{1}{j\omega C}}$$

$$R = \sqrt{\frac{L}{C}} \text{ and } \omega = \frac{1}{\sqrt{LC}}$$

$$\frac{V_o}{V_i} = \frac{\left(\sqrt{\dfrac{L}{C}} + j\sqrt{\dfrac{1}{LC}}\cdot L\right)}{\sqrt{\dfrac{L}{C}} + \dfrac{1}{j\dfrac{1}{\sqrt{LC}}C}}$$

$$= -\frac{\left(\sqrt{\dfrac{L}{C}} + j\sqrt{\dfrac{L}{C}}\right)}{\sqrt{\dfrac{L}{C}} + \dfrac{1}{j}\sqrt{\dfrac{L}{C}}} = -\frac{\left(\sqrt{\dfrac{L}{C}}(1+j)\right)}{\sqrt{\dfrac{L}{C}}\left(1+\dfrac{1}{j}\right)}$$

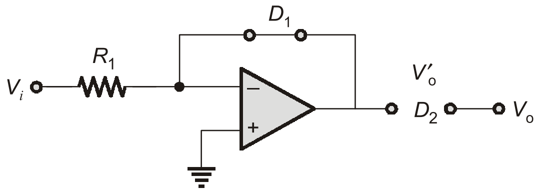
$$V_o = -\frac{(1+j)}{1-j} = -\frac{(\angle 45°)}{(-\angle 45°)}$$

$$= -[45° + 45°] = -90°$$

$$\frac{V_o}{V_i} = -\frac{\pi}{2} = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

1.3 (b) & (c)

Case 1: $V_i > 0 \rightarrow$

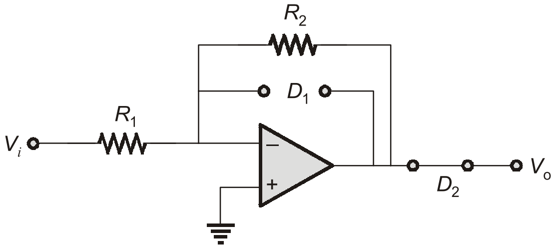


When $V_i > 0$ then V_o' is negative.

So, diode D_1 is forward bias and diode D_2 is reverse bias.

So, V_o = 0 V

Case 2: $V_i < 0 \rightarrow$



When V_i is negative then V_o is positive.

So, diode D_1 is reverse bias and diode D_2 is forward bias

$$V_o = -\frac{R_2}{R_1}V_i$$

1.4 **(d)**

$$\boxed{\frac{1}{\beta} = 1+\frac{R_f}{R_1}} = 1+\frac{8k}{2k} = 1+4 = 5$$

$$\beta = \frac{1}{5} = 0.2$$

$$A\beta = 45 \times 0.2 = 9$$

So, $A\beta$ is not very greater than 1

$$\text{So, } \boxed{A_f = \frac{A}{1+A\beta}} = \frac{45}{1+9} = 4.5$$

1.5 **(c)**

$$A_1 = -\frac{R_f}{R_1} = -\frac{90k}{1k} = -90$$

$$A_2 = \left(1+\frac{R_f}{R_1}\right)\left(\frac{100}{100+1}\right) = 90.09$$

$$A_d = \frac{A_2 - A_1}{2} = \frac{90.09-(-90)}{2} = 90$$

$$A_c = A_1 + A_2 = -90 + 90.09 = 0.09$$

$$\text{CMRR} = \left|\frac{A_d}{A_c}\right| = \frac{90}{0.09} = 1000$$

1.6 **(d)**

The open loop gain of amplifier is very high, so it will act as a comparator.

So, if sinusoidal signal is applied to the input of the high gain comparator, then the comparator generates the square wave output.

1.7 **(*)**

$$\boxed{V_o = \text{Gain}\,(V_{in})}$$

$$V_{\text{in}} = V_{\text{offset voltage}} = 1\text{ mV}$$

$$\boxed{\text{Gain} = 1+\frac{R_f}{R_1}} = 1+\frac{1k}{1k} = 2$$

$$V_o = 2 \times 1 \times 10^{-3} = 2\text{ mV}$$

1.8 **(c)**

Logarithmic amplifier

$$\boxed{I = \frac{V_{in}}{R}}$$

$$\boxed{I_f = I_o e^{V_d/\eta V_T}}$$

$$I = I_f = I_o e^{V_d/\eta V_T}$$

$$\frac{V_{\text{in}}}{R} = I_o e^{V_d/\eta V_T}$$

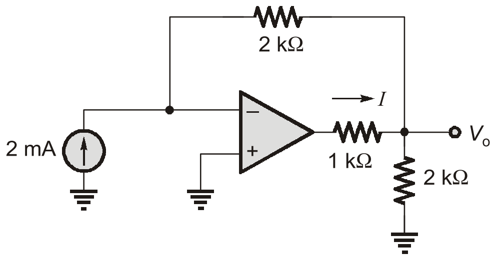
$$e^{V_d/\eta V_T} = \frac{V_{\text{in}}}{RI_o}$$

$$\frac{V_d}{\eta V_T} = ln\left(\frac{V_{\text{in}}}{RI_o}\right)$$

$$V_o = -V_d$$

$$\boxed{V_o = -\eta V_T ln\left(\frac{V_{\text{in}}}{RI_o}\right)}$$

1.9 **(–4)**



$$V_o = -2 \times 10^{-3} \times 2 \times 10^{3} = -4\text{ V}$$

Apply KCL at output node

$$I + 2\text{ mA} = \frac{V_o}{2k} = -\frac{4}{2k} = -2\text{ mA}$$

$$I = -4\text{ mA}$$

1.10 **(0.02)**

$$\boxed{V_o^- = -\frac{R_f}{R_1}V_{\text{in}}} = -\frac{99}{100}\times 4$$

$$= -0.99 \times 4 = -3.96$$

$$V_x = 4\times\frac{100}{100+100} = 2\text{ V}$$

$$\boxed{V_o^+ = \left(1+\frac{R_f}{R_1}\right)V_x} = \left(1+\frac{99}{100}\right)\times 2$$

$$= 1.99 \times 2 = 3.98$$

$$\boxed{V_o = V_o^+ + V_o^-} = 3.98 - 3.96 = 0.02 \text{ V}$$

1.11 Sol.

To increase the stability of op-amps, frequency compensation is used in op-amps.

2. Diodes Applications

2.1 (c)

$$I = \frac{10-6}{50} = \frac{4}{50} = 80 \text{ mA}$$

$$\boxed{I = I_Z + I_L = I_{z\min} + I_{L\max} = I_{z\max} + I_{L\min}}$$

$$I_{z\min} = 5 \text{ mA}$$

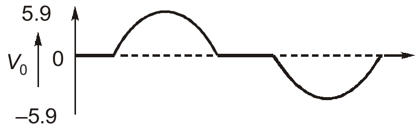
$$80 = 5 + I_{L\max}$$

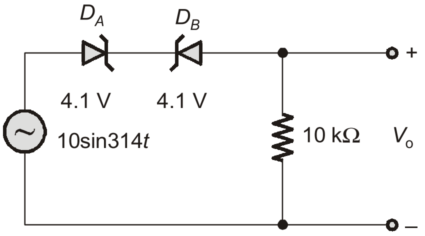
$$I_{L\max} = 75 \text{ mA}$$

$$\boxed{I_{L\max} = \frac{V_L}{R_{\min}}}$$

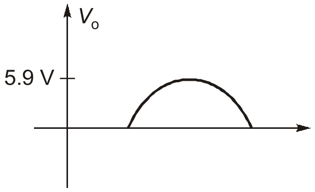
$$R_{\min} = \frac{V_L}{I_{L\max}} = \frac{6}{75 \times 10^{-3}} = 80\ \Omega$$

2.2 (a)

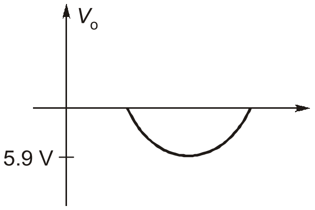




Case 1: During +ve half cycle
Diode D_A is forward bias, so D_A is short circuit.
Diode D_B is reverse bias, so D_B is in conducting state when $V_i > 4.1$ V.



Case 2: During –ve half cycle
Diode D_B is forward bias, so D_B is short circuit.
Diode D_A is reverse bias, so D_A is in conducting state when $|V_i| > 4.1$ V.



3. BJT Analysis

3.1 (b)

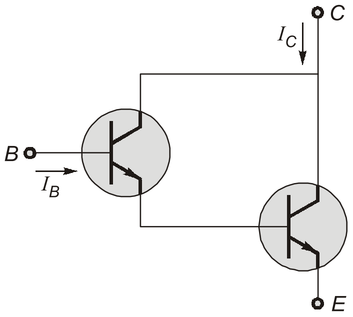
Cascode amplifier is the common emitter followed by common base configuration.

3.2 (c)

$$\boxed{g_m = \frac{I_c}{V_T}}$$

So, if $I_c \uparrow$ then $g_m \uparrow$ $\quad g_m \propto I_c$
So, if the quiescent collector current I_c increases then the transconductance g_m also increases.

3.3 (d)

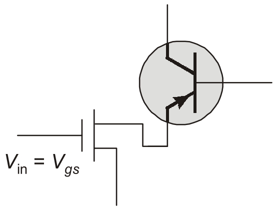


So, overall β of the composite transistor

$$\frac{I_C}{I_B} = \beta = \beta_1 + \beta_2 + \beta_1\,\beta_2$$

$$= 10200$$

3.4 (d)



$$g'_m = \frac{I_c}{V_{gs}}$$

$$I_c = \alpha I_E = \frac{\beta}{1+\beta} I_E$$

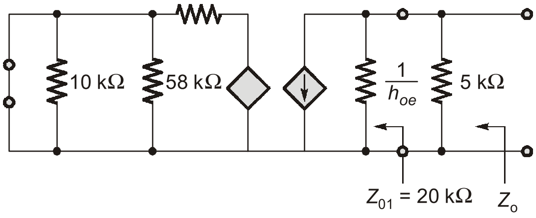
$$g'_m = \frac{\frac{\beta}{1+\beta} I_E}{V_{gs}} = \frac{\beta\, I_E}{(1+\beta)\, V_{gs}}$$

$$I_D = I_E$$

$$g'_m = \frac{\beta}{(1+\beta)} \times \frac{I_D}{V_{gs}} = \frac{\beta}{1+\beta} g_m = \frac{99}{1+99} \times 2 \text{ mA/V}$$

$$g'_m = 1.98 \text{ mA/V}$$

3.5 **(d)**



Output admittance, $Y_o = h_{oe} - \dfrac{h_{fe}\, h_{re}}{h_{ie} + R_s}$

$$= 0.05 \times 10^{-3} - \frac{100 \times 0}{2 \times 10^3 + 10 \times 10^3}$$

$$Y_o = 0.05 \times 10^{-3}$$

$$Z_{01} = \frac{1}{Y_o} = \frac{1}{0.05 \times 10^{-3}} = 20 \text{ k}\Omega$$

Output impedance,

$$Z_0 = Z_{01} \parallel 5 \text{ k} = 20 \text{ k} \parallel 5 \text{ k} = \frac{20 \times 5}{20 + 5}$$

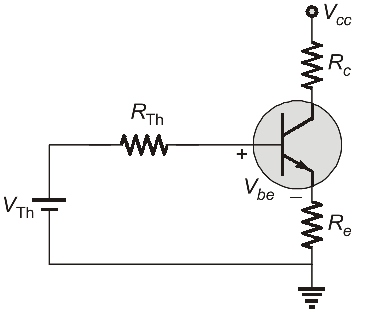
$$Z_0 = 4 \text{ k}\Omega$$

3.6 **(a)**

In the BJT self bias circuit or potential divider circuit provides the maximum bias stability.

3.7 **(b)**

$$\frac{R_E}{R_B} >> 1$$



Simplified self bias circuit using Thevenin theorem

Thevenin open circuit voltage

$$V_{Th} = \frac{V_{cc} R_1}{R_1 + R_2}$$

Thevenin internal resistance

$$R_{Th} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} = R_B$$

Apply KVL to input mesh

$$V_{Th} = I_B R + V_{be} + I_E R_E$$

Put $\quad I_E = I_B + I_C$

$$V_{Th} = I_B R_B + V_{be} + (I_B + I_C) R_E$$

Differentiate w.r.t. I_c, keeping β and V_{be} constant.

$$0 = (R_B + R_E) \frac{\partial I_B}{\partial I_C} + R_E + 0$$

$$\frac{\partial I_B}{\partial I_C} = -\frac{R_E}{R_B + R_E}$$

$$\boxed{S = \frac{1+\beta}{1 - \beta \frac{\partial I_b}{\partial I_c}}}$$

$$\boxed{S = \frac{1+\beta}{1 + \beta \left(\frac{R_E}{R_B + R_E} \right)}}$$

$$\beta >> 1$$

$$\beta \frac{R_E}{R_B + R_E} >> 1$$

$$S = \frac{\beta}{\beta \left(\frac{R_E}{R_B + R_E} \right)}$$

$$\boxed{S = 1 + \frac{R_B}{R_E}}$$

For better stability

$$S \cong 1$$

So, $\quad \dfrac{R_B}{R_E} << 1 \quad ; \quad \dfrac{R_E}{R_B} >> 1$

3.8 **(a) & (b)**

In common base (*CB*) amplifiers input impedance (Z_i) is low and current gain (α) is also low.

$$\alpha = \frac{I_c}{I_e}$$

In common collector (*CC*) amplifiers output impedance (Z_o) is low and current gain (γ) is high

$$\gamma = \frac{I_e}{I_b} = 1 + \beta$$

3.9 **(5)**

Given that β is very large

So, $I_C = I_E$

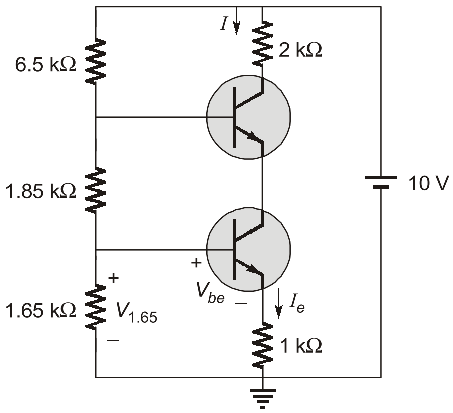
So, the current through 1 kΩ resistance

$$I = 5\text{ mA}$$

$$V_{DC} = 10 - IR = 10 - 5 \times 10^{-3} \times 1 \times 10^{3}$$

$$V_{DC} = 5\text{ V}$$

3.10 **(1)**



Given that β is very large.

So, $I_b = 0$

$$\text{So, } V_{1.65} = 10 \times \frac{1.65}{1.65 + 1.85 + 6.5} = 1.65\text{ V}$$

Apply KVL at input mesh,

$$V_{1.65} = V_{be} + I_e R_e = 0.65 + I_e \times 1\text{ k} = 1.65$$

$$I_e = 1\text{ mA}$$

∵ β is very large,

So, $I_c \cong I_e$

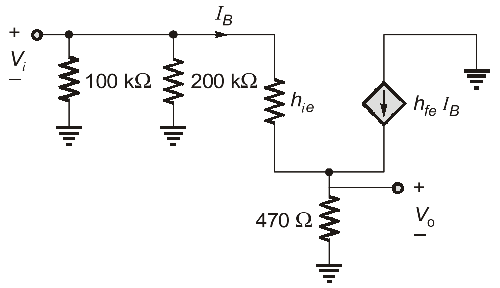
$$I = I_c = 1\text{ mA}$$

3.11 **(c)**

The overall bandwidth of an n-stage turned amplifier is

$$\boxed{BW^* = B\sqrt{2^{1/n} - 1}}$$

3.12 **(c)**



$$A_V = \frac{V_o}{V_i} = \frac{I_B(1 + h_{fe})R_E}{I_B\left[h_{ie} + (1 + h_{fe})\right]R_E} \cong 1$$

3.13 **(d)**

$$f\propto = \frac{DB}{\pi W_B^2}$$

Thus, if W_B will decrease $f\alpha$ will increase.

3.14 **Sol.**

In order to reduce the harmonic distortion in an amplifier its dynamic range has to be **large**.

4. FET and MOSFET Analysis

4.1 **(a)**

$$\boxed{I_{DS} = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2}$$

$$6.4 \times 10^{-3} = 10 \times 10^{-3}\left[1 - \frac{V_{GS}}{(-5)}\right]^2$$

$$0.64 = \left(1 + \frac{V_{GS}}{5}\right)^2$$

$$1 + \frac{V_{GS}}{5} = 0.8$$

$$\frac{V_{GS}}{5} = -0.2$$

$$V_{GS} = -1\text{ V}$$

$$V_{GS} = -I_{DS}R_S$$

$$-1 = -6.4 \times 10^{-3}\,R_s$$

$$R_s = \frac{1}{6.4 \times 10^{-3}} = 156\ \Omega$$

$$R_s \cong 150\ \Omega$$

5. Frequency Response of Amplifier

5.1 **(a)**

The low frequency of operation of a multi-stage RC coupled amplifier is limited by the coupling capacitor.

6. Feedback Amplifiers

6.1 **(c)**

Emitter is output node, it is voltage sampler voltage shunt feedback.

6.2 (a, c)

For identical operational amplifiers, Gain-bandwidth product is constant

$$\text{G.B.W} = \text{Constant}$$

$$A_1 \times BW_1 = A_2 \times BW_2$$

$$BW_2 = \frac{A_1 \times BW_1}{A_2} = \frac{1 \times BW_1}{20} = \frac{BW_2}{20}$$

So, as compare to the unity gain amplifier with gain twenty has less bandwidth.

6.3 (b) & (d)

Negative feedback in amplifiers

(b) improves the signal to noise ratio at the output

(d) reduces distortion.

7. Oscillator Circuits

7.1 Sol.

a-2, b-1, c-3

(a) Hartley oscillator

(2) High frequency oscillator

(b) Wein-bridge oscillator

(1) Low frequency oscillator

(c) Crystal oscillator

(3) Stable frequency oscillator

8. Power Amplifiers

8.1 (b)

Class *A* amplifier ratio

$$= \frac{\text{efficiency of transformer coupled amplifier}}{\text{efficiency of transformer less amplifier}}$$

$$= \frac{50\%}{25\%} = 2$$

8.2 (a, c)

The output of push pull power amplifier consists of only odd harmonics terms.

$$\boxed{I_o = 2k\left[B_1 \cos \omega t + B_3 \cos 3\omega t + B_5 \cos 5\omega t + \cdots\right]}$$

8.3 (c)

$$\frac{P_{D\max}}{P_{AC\max}} = 2$$

For class-A transformer coupled amplifier,

$$P_{D\max} = 2 \times P_{AC\max} = 20 \text{ W}$$

$\therefore$ Power rating $\geq$ 20 W

■■■

Digital Circuits and Computer Organization

UNIT VI

CONTENTS

VI

Digital Circuits and Computer Organization

1. Number Systems

1.1 The subtraction of a binary number *Y* from another binary number *X*, done by adding the 2's complement of *Y* to *X*, results in a binary number without overflow. This implies that the result is:

(a) negative and is in normal form
(b) negative and is in 2's complement form
(c) positive and is in normal form
(d) positive and is in 2's complement form

[1987 : 1 Mark]

1.2 2's complement representation of a 16-bit number (one sign bit and 15 magnitude bits) if FFFF. Its magnitude in decimal representation is

(a) 0
(b) 1
(c) 32, 767
(d) 65, 535

[1993 : 1 Mark]

2. Boolean Algebra

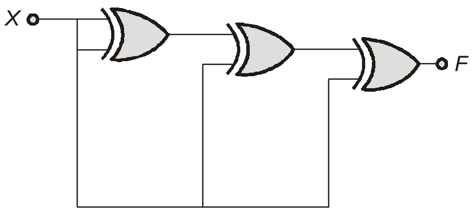
2.1 The number of Boolean functions that can be generated by n variables is equal to:

(a) $2^{2^{n-1}}$
(b) 2^{2^n}
(c) 2^{n-1}
(d) 2^n

[1990 : 1 Mark]

3. Logic Gates

3.1 For the circuit shown below the output *F* is given by



(a) $F = 1$
(b) $F = 0$
(c) $F = X$
(d) $F = \bar{X}$

[1988 : 1 Mark]

3.2 Minimum number of 2-input NAND gates required to implement the function,

$F = (\bar{X} + \bar{Y})(Z + W)$ is

(a) 3
(b) 4
(c) 5
(d) 6

[1988 : 1 Mark]

3.3 Indicate which of the following logic gates can be used to realized all possible combinational Logic functions:

(a) OR gates only
(b) NAND gates only
(c) EX-OR gates only
(d) NOR gates only

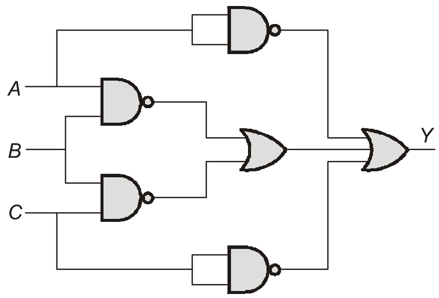
[1989 : 1 Mark]

3.4 Boolean expression for the output of XNOR (equivalence) logic gate with inputs *A* and *B* is

(a) $A\bar{B} + \bar{A}B$
(b) $\bar{A}\bar{B} + AB$
(c) $(\bar{A} + B)(A + \bar{B})$
(d) $(\bar{A} + \bar{B})(A + B)$

[1993 : 1 Mark]

3.5 For the logic circuit shown in Figure, the output is equal to



(a) $\overline{ABC}$
(b) $\bar{A} + \bar{B} + \bar{C}$
(c) $\overline{AB} + \overline{BC} + \bar{A} + \bar{C}$
(d) $\overline{AB} + \overline{BC}$

[1993 : 1 Mark]

3.6 The output of a logic gate is '1' when all its a inputs are at logic '0'. The gate is either

(a) a NAND or an EX-OR gate
(b) a NOR or an EX-NOR gate
(c) an OR or an EX-NOR gate
(d) an AND or an EX-OR gate

[1994 : 1 Mark]

3.7 A ring oscillator consisting of 5 inverters is running at a frequency of 1.0 MHz. The propagation delay per gate is ______ n sec.

[1994 : 1 Mark]

4. Combinational Circuits

4.1 The minimal function that can detect a “divisible by 3” 8421 BCD code digit (representation is $D_8\, D_4\, D_2\, D_1$) is given by

(a) $D_8\, D_1 + D_4\, D_2 + \overline{D_8}D_2D_1$

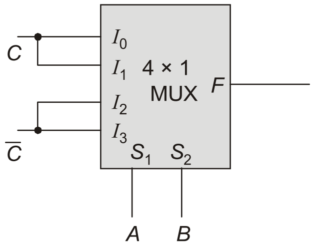
(b) $D_8D_1 + D_4D_2\bar{D}_1 + \bar{D}_4D_2D_1 + \bar{D}_8\bar{D}_4\bar{D}_2\bar{D}_1$

(c) $D_8D_1 + D_4D_2 + \bar{D}_8\bar{D}_4\bar{D}_2\bar{D}_1$

(d) $D_4D_2\bar{D}_1 + D_4D_2D_1 + D_8\bar{D}_4D_2D_1$

[1990 : 1 Mark]

4.2 The logic realized by the circuit shown in figure is

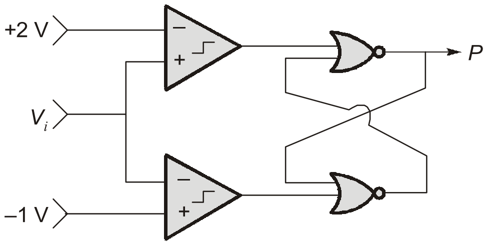


(a) $F = A \odot C$
(b) $F = A \oplus C$
(c) $F = B \odot C$
(d) $F = B \oplus C$

[1992 : 1 Mark]

5. Sequential Circuits

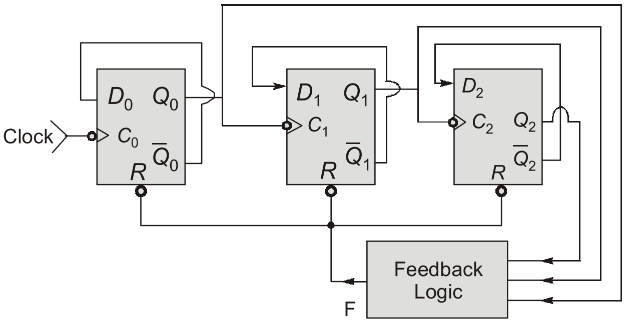
5.1 Choose the correct statements relating to the circuit of figure



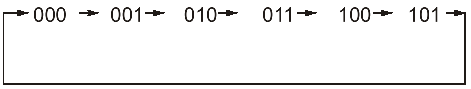
(a) For $V_i = -2$ V, $P = 0$
(b) For $V_i = +3$ V, $P = 0$
(c) For $V_i = 0$ V, $P = 0$ always
(d) For $V_i = 0$ V, P can be either 0 or 1.

[1987 : 1 Mark]

5.2 A ripple counter using negative edge-triggered *D*-flip flops is shown in figure below. The flip-flops are cleared to ‘0’ at the *R* input. The feedback logic is a to be designed to obtain the count sequence shown in the same figure. The correct feedback logic is:



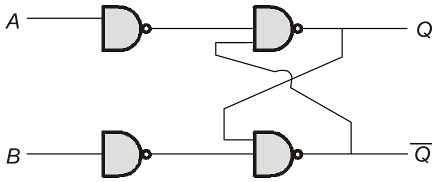
Count sequence in the order of $Q_2Q_1Q_0$:



(a) $F = \overline{Q_2\, Q_1\, \bar{Q}_0}$
(b) $F = Q_2\, \bar{Q}_1\, \bar{Q}_0$
(c) $F = \bar{Q}_2\, \bar{Q}_1\, Q_0$
(d) $F = \bar{Q}_2\, \bar{Q}_1\, \bar{Q}_0$

[1987 : 1 Mark]

5.3 The circuit given below is a



(a) *J-K* Flip-flop
(b) Johnson’s counter
(c) *R-S* latch
(d) None of above.

[1988 : 1 Mark]

5.4 A 4-bit modulo-16 ripple counter uses *JK* flip-flops. If the propagation delay of each *FF* is 50 ns, the maximum clock frequency that can be used is equal to:

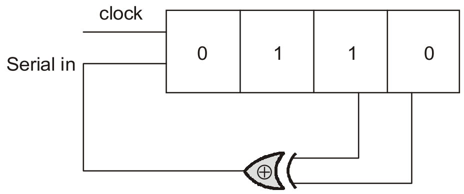
(a) 20 MHz
(b) 10 MHz
(c) 5 MHz
(d) 4 MHz

[1990 : 1 Mark]

5.5 An *S-R* FLIP-FLOP can be converted into a *T* flip-flop by connecting to ______ *Q* and ____ to $\bar{Q}$.

[1991 : 1 Mark]

5.6 The initial contents of the 4-bit serial-in-parallel-out, right-shift, Shift Register shown in the figure is 0110. After three clock pulses are applied, the contents of the Shift Register will be



(a) 0000
(b) 0101
(c) 1010
(d) 1111

[1992 : 1 Mark]

5.7 A pulse train with a frequency of 1 MHz is counted using a modulo-1024 ripple-counter built with *J-K* flip flops. For proper operation of the counter. The maximum permissible propagation delay per flip flop stage is ________ n sec.

[1993 : 1 Mark]

5.8 Synchronous counters are ________ than the ripple counters.

[1994 : 1 Mark]

6. Logic Families

6.1 Fill in the blanks of the statements below concerning the following Logic Families :

Standard TTL (74 XXLL), Low power TTL (74L XX) Low power schottky

TTL(74L SXX), Schottky TTL(74 SXX), Emitter coupled Logic (ECL), CMOS

(a) Among the TTL Families, __________ family requires considerably less power than the standard TTL (74 XX) and also has comparable propagation delay.

(b) Only the __________ family can operate over a wide range of power supply voltages

[1987 : 1 Mark]

6.2 Given that for a logic family,

V_{OH} is the minimum output high-level voltage.

V_{OL} is the maximum output-low-level voltage.

V_{IH} is the minimum acceptable input high-level voltage and

V_{IL} is the maximum acceptable input low-level voltage,

The correct relationship is

(a) $V_{IH} > V_{OH} > V_{IL} > V_{OL}$

(b) $V_{OH} > V_{IH} > V_{IL} > V_{OL}$

(c) $V_{IH} > V_{OH} > V_{OL} > V_{IL}$

(d) $V_{OH} > V_{IH} > V_{OL} > V_{IL}$ [1987 : 2 Marks]

6.3 Among the digital IC-families-ECL, TTL and CMOS:

(a) ECL has the least propagation delay

(b) TTL has the largest fan-out

(c) CMOS has the biggest noise margin

(d) TTL has the lowest power consumption

[1989 : 1 Mark]

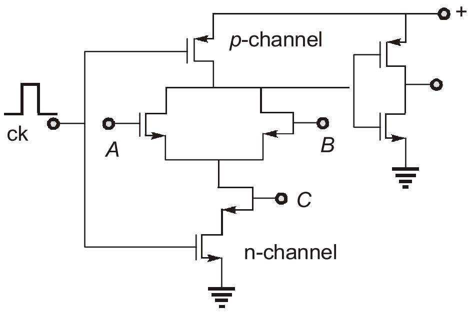
6.4 A logic family has threshold voltage V_R = 2 V, minimum guaranteed output high voltage V_{OH} = 4 V, minimum accepted input high voltage V_{IH} = 3 V, maximum guaranteed output low voltage V_{OL} = 1 V, and maximum accepted input low voltage V_{IL} = 1.5 V. Its noise margin is

(a) 2 V (b) 1 V

(c) 1.5 V (d) 0.5 V

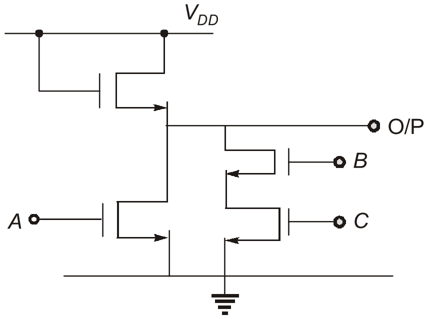
[1989 : 1 Mark]

6.5 In figure, the Boolean expression for the output in terms of inputs *A*, *B* and *C* when the clock '*ck*' is high, is given by



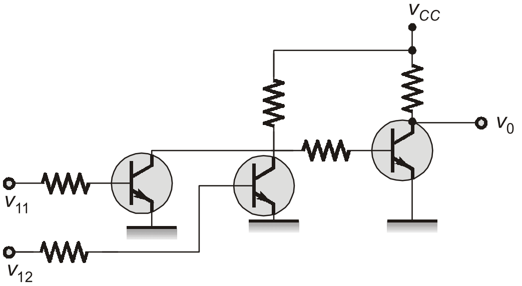
[1991 : 1 Mark]

6.6 The CMOS equivalent of the following *n* MOS gate (figure) is (draw the circuit).



[1991 : 1 Mark]

6.7 Figure shows the circuit of a gate in the Resistor Transistor Logic (RTL) family. The circuit represents a



(a) NAND (b) AND

(c) NOR (d) OR

[1992 : 1 Mark]

6.8 In the output stage of a standard TTL, we have a diode between the emitter of the pull-up transistor and the collector of the pull-down transistor. The purpose of this diode is to isolate the output node from the power supply V_{CC}.
[1994 : 2 Marks]

7. Memories

7.1 Choose the correct statement from the following:
(a) PROM contains a programmable AND array and a fixed OR array.
(b) PLA contains a fixed AND array and a programmable OR array.
(c) PROM contains a fixed AND array and a programmable OR array
(d) PLA contains a programmable AND array and a programmable OR array. [1992 : 1 Mark]

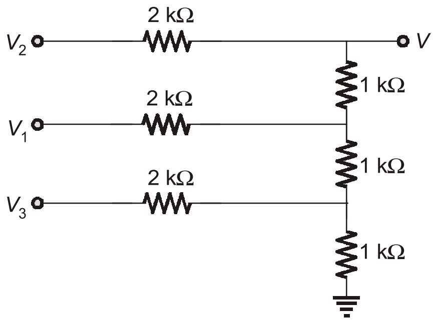
7.2 A PLA can be
(a) as a microprocessor
(b) as a dynamic memory
(c) to realize a sequential logic
(d) to realize a combinational logic.
[1994 : 1 Mark]

7.3 A dynamic RAM consists of
(a) 6 transistors
(b) 2 transistors and 2 capacitors
(c) 1 transistor and 1 capacitor
(d) 2 capacitors only [1994 : 1 Mark]

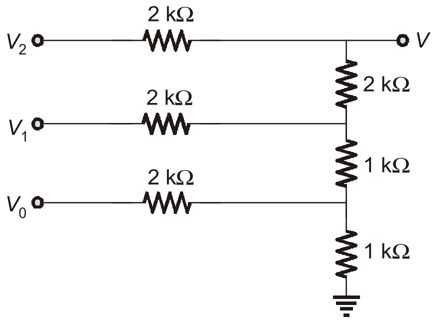
8. ADC and DAC

8.1 Which of the resistance networks of figure can be used as 3 bit R-$2R$ ladder DAC. Assume V_0 corresponds to LSB.

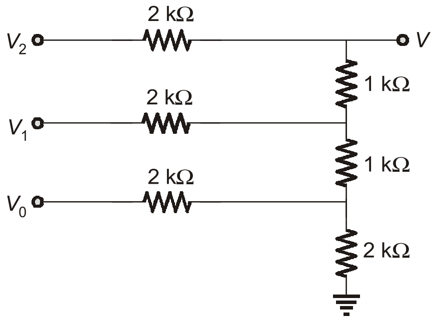
(i)



(ii)



(iii)



(a) Both (i) and (ii) (b) Both (i) and (ii)
(c) Only (iii) (d) Only (ii)
[1990 : 1 Mark]

8.2 (a) Successive approximation
(b) Dual-slope
(b) Parallel Comparator
Maximum conversion time for 8 bit ADC in clock cycles
(1) 1 (2) 2 (3) 16
(4) 256 (5) 512
[1994 : 1 Mark]

■■■

Answers Digital Circuits and Computer Organization

1.1	(b, c)	1.2	(b)	2.1	(b)	3.1	(b)	3.2	(b)	3.3	(b, d)	3.4	(b, c)	3.5	(b)	3.6	(b)
3.7	(100)	4.1	(b)	4.2	(b)	4.3	(b)	4.4	(c)	5.1	(b)	5.2	(a)	5.3	(c)	5.4	(c)
5.5	(sol.)	5.6	(c)	5.7	(100)	5.8	(sol.)	6.1	(a, b)	6.2	(b)	6.3	(a)	6.4	(d)	6.5	(sol.)
6.6	(sol.)	6.7	(d)	6.8	(sol.)	7.1	(c, d)	7.2	(d)	7.3	(c)	8.1	(c)				

Explanations Digital Circuits and Computer Organization

1. Number Systems

1.1 **(b) & (c)**

Negative and is in 2's complement form.
Positive and is in normal form

1.2 **(b)**

1

F F F F

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

2's complement

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1

= 0 0 0 1

= 1

2. Boolean Algebra

2.1 **(b)**

Boolean functions possible are 2^{2^n}

3. Logic Gates

3.1 **(b)**

Output of 1st EX-OR gate

$$F_1 = X \oplus X = 0$$

Output of 2nd EX-OR gate

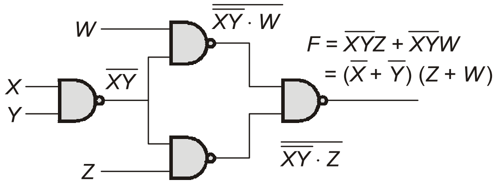
$$F_2 = X \oplus 0 = X$$

Output of 3rd EX-OR gate

$$F = X \oplus X = 0$$

3.2 **(b)**

$$F = (\bar{X}+\bar{Y})(Z+W) = (\overline{XY})\cdot(Z+W) = \overline{XYZ}\cdot\overline{XYW}$$



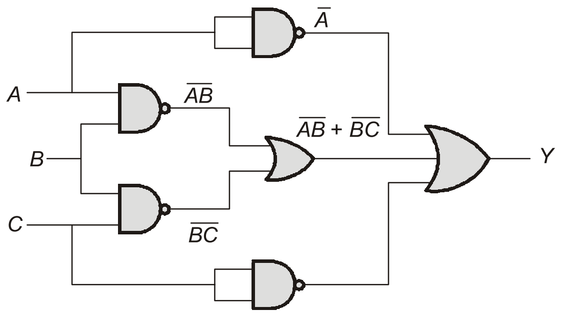
3.3 **(b) & (d)**

NAND and NOR gates can be used to realize all possible combinational logic functions.

3.4 **(b) & (c)**

$\bar{A}\bar{B} + AB$ and $(\bar{A}+B)(A+\bar{B})$

3.5 **(b)**



$$Y = \bar{A} + \overline{AB} + \overline{BC} + \bar{C}$$

$$Y = \bar{A} + (\bar{A}+\bar{B}) + (\bar{B}+\bar{C}) + \bar{C}$$

$$Y = \bar{A} + \bar{B} + \bar{C}$$

3.6 **(b)**

NOR gate → $F = \overline{A+B} = \bar{A}\bar{B}$

EX-NOR gate → $F = A \odot B = \bar{A}\bar{B} + AB$

3.7 **(100)**

$$f_{\text{clk}} = \frac{1}{2Nt_{pdFF}}$$

$$t_{pdFF} = \frac{1}{2Nf_{\text{clk}}} = \frac{1}{2\times 5\times 10^6}$$

$$t_{pdFF} = 100 \text{ nsec}$$

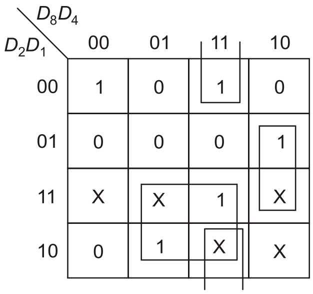
4. Combinational Circuits

4.1 (b)

Truth table:

	D_8	D_4	D_2	D_1	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1

K-Map:



D_2D_1 \ D_8D_4	00	01	11	10
00	1	0	1	0
01	0	0	0	1
11	X	X	1	X
10	0	1	X	X

$$Y = D_8D_1 + D_4D_2\bar{D}_1 + \bar{D}_4D_2D_1 + \bar{D}_8\bar{D}_4\bar{D}_2\bar{D}_1$$

4.2 (b)

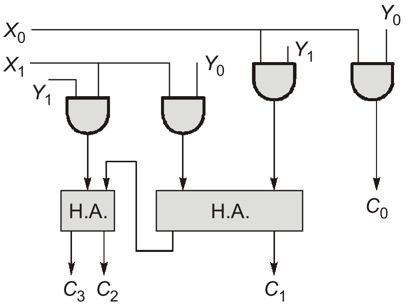
$$F = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$$

$$F = \bar{A}C(\bar{B}+B) + A\bar{C}(\bar{B}+B)$$

$$F = \bar{A}C + A\bar{C} = A \oplus C$$

4.3 (b)

Two bit binary multiplier



$$\begin{array}{cccc} & & X_1 & X_0 \\ & & Y_1 & Y_0 \\ \hline & & X_1Y_0 & X_0Y_0 \\ & X_1Y_1 & Y_1X_0 & \downarrow \\ \hline C_3 & C_2 & C_1 & C_0 \end{array}$$

4.4 (c)

A	B	D (difference)	X (borrow)
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = A \oplus B = \bar{A}B + A\bar{B}$$

$$X = \bar{A}B$$

5. Sequential Circuits

5.1 (b)

For $V_i = +3$ V, $P = 0$

When $V_i = +3$ V

Output of comparator 1 = logic 1

Output of comparator 2 = logic 0

$$P = \overline{1 + \text{(output of 2nd NOR gate)}}$$

$$P = \bar{1}$$

$$P = 0$$

5.2 (a)

$$D_0 = \bar{Q}_0$$

$$D_1 = \bar{Q}_1$$

$$D_2 = \bar{Q}_2$$

So, flip-flop will be cleared at

$$\text{NAND gate} \rightarrow \frac{1 \quad 1 \quad 0}{Q_2 \;\; Q_1 \;\; \bar{Q}_0}$$

5.3 (c)

R-S latch.

5.4 (c)

$$f_{\text{clk}} = \frac{1}{nt_{pdff}}$$

$$= \frac{1}{4 \times 50 \times 10^{-9}} \qquad \begin{Bmatrix} 2^4 = 16 \\ n = 4 \end{Bmatrix}$$

$$f_{\text{clk}} = 5 \text{ MHz}$$

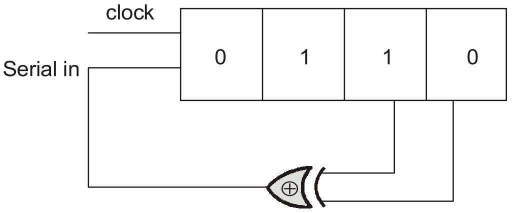
5.5 Sol.

$$S = T\bar{Q}_n$$

$$R = TQ_n$$

S-R flip-flop can be converted into *T*-flip-flop by connecting R to Q_n and S to $\bar{Q}_n$.

5.6 **(c)**



Serial in = $D_1 \oplus D_0$

	0	1	1	0
1st clk	1	0	1	1
2nd clk	0	1	0	1
3rd clk	1	0	1	0

5.7 **(100)**

Ripple counter → n-states

$$n = \log_2(1024) = 10$$

$$t_{pd\,ff} = \frac{1}{nf_{\text{clk}}} = \frac{1}{10 \times 10^6} = 100\ \text{n-sec}$$

5.8 **Sol.**

Synchronous counters are faster than ripple counters.

6. Logic Families

6.1 **(a) & (b)**

74LS and CMOS.

6.2 **(b)**

$V_{OH} > V_{IH} > V_{IL} > V_{OL}$.

6.3 **(a)**

ECL is the fastest logic family.

6.4 **(d)**

$$(NM)_H = V_{OH} - V_{IH} = 4 - 3 = 1\ \text{V}$$
$$(NM)_L = V_{IL} - V_{OL} = 1.5 - 1 = 0.5\ \text{V}$$

Noise margin = $\min \text{ of } [(NM)_H, (NM)_L]$

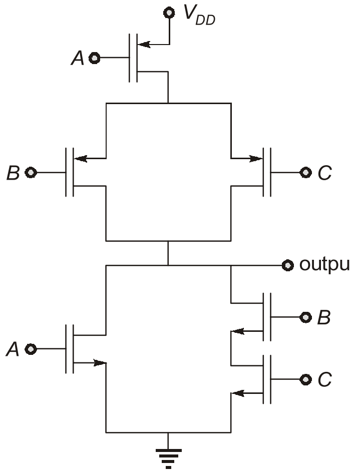
= 0.5 V

6.5 **Sol.**

When clock is high then p-channel is off, so the input to CMOS is logic 0, then the output of the CMOS inverter is logic 1.

6.6 **Sol.**

CMOS circuit: PMOS and NMOS both



Output = $\overline{A + BC}$

6.7 **(d)**

Output V_o will be low when all the inputs are low. If any of the input is/are active high then the output will be high. So, the given circuit represents the OR gate.

6.8 **Sol.**

The purpose of keeping a diode is the output of TTL circuit is to keep pull-up transistor in the OFF state as long as pull-down transistor is the ON state.

7. Memories

7.1 **(c, d)**

PROM contains a fixed AND array and a programmable OR array
PLA contains a programmable AND array and a programmable OR array.

7.2 **(d)**

PLA is a type of fixed architecture logic devices with programmable AND gates followed by programmable OR gates. The PLA can be used to implement a complex combinational circuits.

7.3 **(c)**

A dynamic RAM consists of 1 transistor and 1 capacitor or 2 MOSFETs.

8. ADC and DAC

8.1 **(c)**

R-$2R$ ladder DAC has R with ground & $2R$ with input.

■■■

Signals and Systems

UNIT VII

CONTENTS

VII

Signals and Systems

1. Basics of Signals and Systems

1.1 An excitation is applied to a system at $t = T$ and its response is zero for $-\infty < t < T$. Such a system is a

(a) non-causal system (b) stable system

(c) causal system (d) unstable system

[1991 : 2 Marks]

1.2 Which of the following signals is/are periodic?

(a) $s(t) = \cos 2t + \cos 3t + \cos 5t$

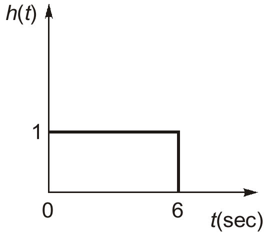
(b) $s(t) = \exp(j8\pi t)$

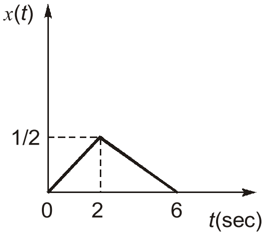
(c) $s(t) = \exp(-7t)\sin 10\pi t$

(d) $s(t) = \cos 2t \cos 4t$ [1992 : 2 Marks]

2. LTI Systems Continuous and Discrete (Time Domain)

2.1 The impulse response and the excitation function of a linear time invariant causal system are shown in figure (a) and (b) respectively. The output of the system at $t = 2$ sec. is equal to





(a) 0 (b) 1/2

(c) 3/2 (d) 1 [1990 : 2 Marks]

3. Fourier Series

3.1 A half-wave rectified sinusoidal waveform has a peak voltage of 10 V. Its average value and the peak value of the fundamental component are respectively given by

(a) $\frac{20}{\pi}$ V, $\frac{10}{\sqrt{2}}$ V (b) $\frac{10}{\pi}$ V, $\frac{10}{\sqrt{2}}$ V

(c) $\frac{10}{\pi}$ V, 5 V (d) $\frac{20}{\pi}$ V, 5 V

[1987 : 2 Marks]

3.2 Fourier series of the periodic function (period 2π) defined by

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

is $\frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi n^2}[\cos(n\pi) - 1]\cos(nx) - \frac{1}{n}\cos(n\pi)\sin(nx)\right].$

By putting $x = \pi$ in the above, one can deduce that the sum of the series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$ is

[1993 : 2 Marks]

3.3 The Fourier series of an odd periodic function, contains only

(a) odd harmonics (b) even harmonics

(c) cosine terms (d) sine terms

[1994 : 1 Mark]

4. Fourier Transforms , Frequency Response and Correlation

4.1 Specify the filter type if its voltage transfer function $H(s)$ is given by

$$H(s) = \frac{K(s^2 + \omega_0^2)}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

(a) all pass filter (b) low pass filter

(c) band pass filter (d) notch filter

[1988 : 2 Marks]

4.2 The magnitude and phase functions for a distortionless filter should respectively be

	(Magnitude)	**(Phase)**
(a)	Linear	Constant
(b)	Constant	Constant
(c)	Constant	Linear
(d)	Linear	Linear

[1990 : 2 Marks]

4.3 If $G(f)$ represents the Fourier transform of a signal $g(t)$ which is real and odd symmetric in time, then

(a) $G(f)$ is complex

(b) $G(f)$ is imaginary

(c) $G(f)$ is real

(d) $G(f)$ is real and non-negative

[1992 : 2 Marks]

5. Laplace Transform

5.1 Laplace transforms of the functions $tu(t)$ and $u(t)\sin(t)$ are respectively:

(a) $\frac{1}{s^2}, \frac{s}{s^2+1}$
(b) $\frac{1}{s}, \frac{1}{s^2+1}$
(c) $\frac{1}{s^2}, \frac{1}{s^2+1}$
(d) $s, \frac{s}{s^2+1}$

[1987 : 2 Marks]

5.2 The Laplace transform of a function $f(t)\,u(t)$, where $f(t)$ is periodic with period T, is $A(s)$ times the Laplace transform of its first period. Then

(a) $A(s) = s$
(b) $A(s) = 1/(1 - \exp(-Ts))$
(c) $A(s) = 1/(1 + \exp(-Ts))$
(d) $A(s) = \exp(Ts)$

[1988 : 2 Marks]

5.3 The transfer function of a zero-order hold is

(a) $\frac{1-\exp(-Ts)}{s}$
(b) $1/s$
(c) 1
(d) $\frac{1}{[1-\exp(-Ts)]}$

[1988 : 2 Marks]

5.4 The transfer function of a zero-order hold is

(a) $\frac{1-\exp(-Ts)}{s}$
(b) $1/s$
(c) 1
(d) $1/[-\exp(-Ts)]$

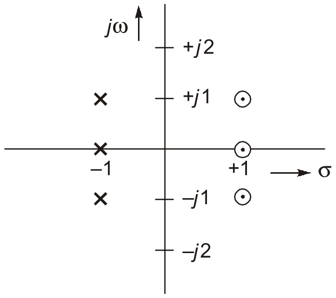
[1988 : 2 Marks]

5.5 The response of an initially relaxed linear constant parameter network to a unit impulse applied at $t = 0$ is $4e^{-2t}u(t)$. The response of this network to a unit step function will be

(a) $2[1 - e^{-2t}]u(t)$
(b) $4[e^{-t} - e^{-2t}]u(t)$
(c) $\sin 2t$
(d) $(1 - 4e^{-4t})\,u(t)$

[1990 : 2 Marks]

5.6 The pole-zero pattern of a certain filter is shown in figure. The filter must be of the following type



(a) low-pass
(b) high-pass
(c) all-pass
(d) band-pass

[1991 : 2 Marks]

5.7 The voltage across an impedance in a network is $V(s) = Z(s)\,I(s)$, where $V(s)$, $Z(s)$ and $I(s)$ are the Laplace transform of the corresponding time functions $v(t)$, $z(t)$ and $i(t)$. The voltage $v(t)$ is

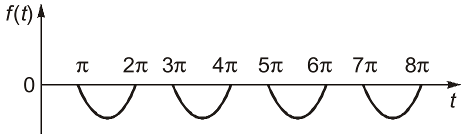
(a) $v(t) = z(t) \cdot i(t)$
(b) $v(t) = \int_0^t i(\tau)\,z(t-\tau)\,d\tau$
(c) $v(t) = \int_0^t i(\tau)\,z(t+\tau)\,d\tau$
(d) $v(t) = z(t) + i(t)$

[1991 : 2 Marks]

5.8 The Laplace transform of the periodic function $f(t)$ described by the curve below, i.e.

$$f(t) = \begin{cases} \sin t & \text{if } (2n-1)\pi \le t \le 2n\pi \quad (n = 1, 2, 3, \cdots) \\ 0 & \text{otherwise} \end{cases}$$

is ________.



[1993 : 2 Marks]

5.9 If $F(s) = L[f(t)] = \frac{K}{(s+1)(s^2+4)}$ then $\lim_{t\to\infty} f(t)$ is given by

(a) K/4
(b) zero
(c) infinite
(d) undefined

[1993 : 2 Marks]

5.10 The Laplace transform of a unit ramp function starting at $t = a$, is

(a) $\frac{1}{(s+a)^2}$
(b) $\frac{e^{-as}}{(s+a)^2}$
(c) $\frac{e^{-as}}{s^2}$
(d) $\frac{a}{s^2}$

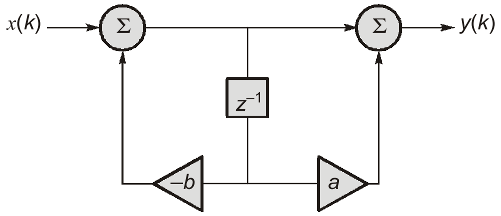
[1994 : 1 Mark]

5.11 Indicate whether the following statement is TRUE/FALSE, Give reason for your answer. If $G(s)$ is a stable transfer function, the $F(s) = \frac{1}{G(s)}$ is always a stable transfer function.

[1994 : 1 Mark]

6. Z-Transform

6.1 Consider the system shown in the figure below. The transfer function $Y(z)/X(z)$ of the system is



(a) $\frac{1+az^{-1}}{1+bz^{-1}}$ (b) $\frac{1+bz^{-1}}{1+az^{-1}}$

(c) $\frac{1+az^{-1}}{1-bz^{-1}}$ (d) $\frac{1-bz^{-1}}{1+az^{-1}}$

[1988 : 2 Marks]

6.2 The z-transform of the following real exponential sequence

$x(nT) = a^n,\ nT \geq 0$

$= 0,\ nT < 0,\ a > 0$

(a) $\frac{1}{1-z^{-1}}; |z| > 1$ (b) $\frac{1}{1-az^{-1}}; |z| > a$

(c) 1 for all z (d) $\frac{1}{1-az^{-1}}; |z| < a$

[1990 : 2 Marks]

6.3 A linear discrete-time system has the characteristics equation, $z^3 - 0.81z = 0$. The system

(a) is stable

(b) is marginally stable

(c) is unstable

(d) stability cannot be assessed from the given information

[1992 : 2 Marks]

7. Sampling

7.1 A signal containing only two frequency components (3 kHz and 6 kHz) is sampled at the rate of 8 kHz, and then passed through a low pass filter with a cut-off frequency of 8 kHz. The filter output

(a) is an undistorted version of the original signal

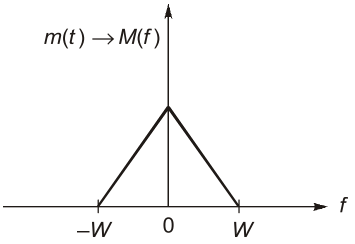
(b) contains only the 3 kHz component

(c) contains the 3 kHz component and a spurious component of 2 kHz

(d) contains both the components of the original signal and two spurious components of 2 kHz and 5 kHz.

[1988 : 2 Marks]

7.2 Increased pulse-width in the flat-top sampling, leads to



(a) attenuation of high frequencies in reproduction

(b) attenuation of low frequencies in reproduction

(c) greater aliasing errors in reproduction

(d) no harmful effects in reproduction

[1994 : 1 Mark]

■■■

Answers Signals and Systems

1.1	(c)	1.2	(a, b, d)	2.1	(b)	3.1	(c)	3.2	(1.23)	3.3	(d)	4.1	(d)
4.2	(c)	4.3	(b)	5.1	(c)	5.2	(b)	5.3	(a)	5.4	(a)	5.5	(a)
5.6	(c)	5.7	(b)	5.8	(sol.)	5.9	(d)	5.10	(c)	5.11	(sol.)	6.1	(a)
6.2	(b)	6.3	(a)	7.1	(d)	7.2	(a)						

Explanations Signals and Systems

1. Basics of Signals and Systems

1.1 (c)

For the given system if the response is zero prior to the application of the excitation. Then such a system is called causal system.

1.2 (a, b, d)

(a) $s(t)$ is periodic as the ratio of any two frequencies $= \dfrac{p}{q}$ is rational

where p and q are integers.

(b) $s(t)$ is periodic with $\omega = 8\pi$

(d) $2\cos A \cos B = \cos(A - B) + \cos(A + B)$

$$s(t) = \frac{1}{2}[\cos 2t + \cos 6t]$$

So, $s(t)$ is periodic with fundamental frequency 2 rad/sec.

2. LTI Systems Continuous and Discrete (Time Domain)

2.1 (b)

For causal LTI system

$$h(t) = 0 \quad \text{for } t < 0$$

$$y(t) = x(t) * h(t)$$

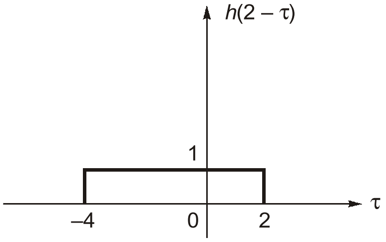
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

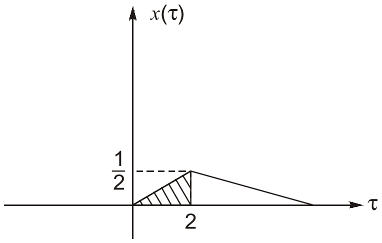
$$y(t) = \int_{0}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

as $x(t) = 0 \quad t < 0$

and $h(t) = 0 \quad t < 0$

$$y(2) = \int_{0}^{2} x(\tau)\, h(2-\tau)\, d\tau$$



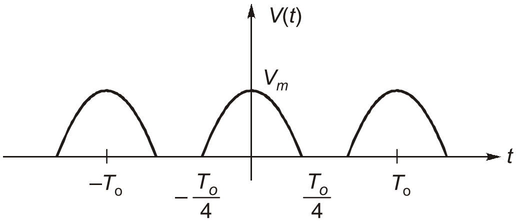


So, $y(2) = \dfrac{1}{2} \times \left(2 \times \dfrac{1}{2}\right)$

$$y(2) = \frac{1}{2}$$

3. Fourier Series

3.1 (c)



$$V(t) = V_m \cos(\omega_o t) \qquad |t| \le \frac{T_o}{4}$$

Time period $= T_o$

$$f_o = \frac{1}{T_o} = \text{fundamental frequency}$$

$$= 1^{\text{st}} \text{ harmonics}$$

Fourier series of $v(t)$ is given by

$$v(t) = a_0 + a_1\cos(\omega_o t) + a_2\cos(2\omega_o t) + \cdots$$

$$a_0 = \text{dc value} = \text{average value}$$

$$a_0 = \frac{1}{T_0}\int_{T_0} v(t)dt$$

$$= \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} V_m \cos(\omega_o t)dt$$

$$a_0 = \frac{1}{T_o}\frac{V_m[\sin\omega_o t]_{-T_o/4}^{T_o/4}}{\omega_o}$$

$$a_0 = \frac{V_m}{T_o\frac{2\pi}{T_o}}\left[\sin\left(\frac{2\pi}{T_o}\cdot\frac{T_o}{4}\right) - \sin\left\{\frac{2\pi}{T_o}\left(-\frac{T_o}{4}\right)\right\}\right]$$

$$a_0 = \frac{V_m}{2\pi}\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{V_m}{2\pi}[1-(-1)] = \frac{V_m}{2\pi} \times 2$$

$$a_0 = \frac{V_m}{\pi}$$

So, average value $= a_0 = \frac{V_m}{\pi} = \frac{10}{\pi}\text{V}$

$$a_1 = \frac{2}{T_0}\int_{T_0} v(t)\cos(\omega_o t)dt$$

$$a_1 = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} V_m\cos(\omega_o t)\cos(\omega_o t)dt$$

$$a_1 = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} V_m\cos^2(\omega_o t)dt$$

$$a_1 = \frac{2V_m}{T_o}\int_{-T_o/4}^{T_o/4}\frac{(1+\cos 2\omega_o t)}{2}dt$$

$$a_1 = \frac{2V_m}{2T_o}\left[\int_{-T_o/4}^{T_o/4} dt + \int_{-T_o/4}^{T_o/4}\cos 2\omega_o t dt\right]$$

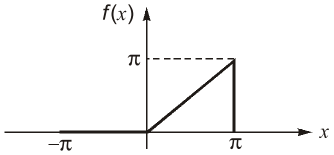
$$a_1 = \frac{V_m}{T_o}\times\left[\frac{T_o}{4}-\left(-\frac{T_o}{4}\right)\right] = \frac{V_m}{T_o}\times\frac{T_o}{2}$$

$$a_1 = \frac{V_m}{2}$$

$$V_m = 10$$

So, $a_1 = \frac{10}{2} = 5\,\text{V}$

3.2 (1.23)



Time period $= T = \pi - (-\pi) = 2\pi$

At the point of discontinuity $x = \pi$

$f(x)$ expressed in Fourier series converge to the middle value $= \frac{\pi}{2}$.

From the given trigonometric form of Fourier series, at $x = \pi$.

$$f(\pi) = \frac{\pi}{4} + \frac{2}{\pi}s = \frac{\pi}{2}$$

where, $s = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

$$\frac{2}{\pi}s = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$s = \frac{\pi^2}{8}$$

3.3 (d)

The Fourier series of an odd periodic function does not contain the dc term ($a_o = 0$) and cosine terms. The Fourier series of an odd periodic function contains only sine terms.

4. Fourier Transforms , Frequency Response and Correlation

4.1 (d)

Given that,

$$H(s) = \frac{K(s^2+\omega_0^2)}{s^2+\left(\frac{\omega_0}{Q}\right)s+\omega_0^2}$$

$$s = j\omega$$

$$H(0) = \frac{K(0+\omega_0^2)}{0+0+\omega_0^2} = K$$

$$H(0) \neq 0$$

$$H(\infty) = \frac{Ks^2\left(1+\frac{\omega_0^2}{s}\right)}{s^2\left(1+\left(\frac{\omega_0}{Q}\right)\frac{1}{s}+\frac{\omega_0^2}{s^2}\right)}$$

$$H(\infty) = \frac{K(1+0)}{1+0+0} = K$$

$$H(\infty) \neq 0$$

Here the given transfer function is a notch filter.

4.2 (c)

For distortionless transmission

$|H(f)| =$ constant and

$\angle H(f) = -kf$, where k is a constant

$H(f)$ is the transfer function.

So, the magnitude function of a distortionless filter should be constant.

The phase function of a distortionless filter should be linear.

4.3 (b)

Fourier transform of real and odd symmetric signal is imaginary and odd function of frequency.

5. Laplace Transform

5.1 **(c)**

$$tu(t) \xleftrightarrow{\text{L.T.}} \frac{1}{s^2}$$

$$u(t)\sin at \xleftrightarrow{\text{L.T.}} \frac{a}{s^2+a^2}$$

$$u(t)\sin t \xleftrightarrow{\text{L.T.}} \frac{1}{s^2+1^2} = \frac{1}{s^2+1}$$

5.2 **(b)**

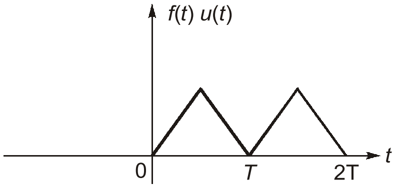
The given function represents a causal periodic signal.

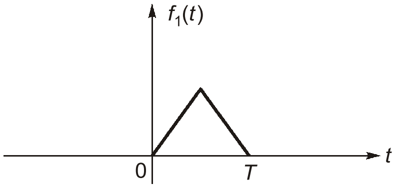
$$f(t)u(t) = 0 \qquad t<0$$

$$\text{Period of } f(t) = T \qquad \text{for } t>0$$

Let $$f_1(t) = f(t)u(t) \qquad 0 \le t \le T$$

$$= 0 \qquad \text{otherwise}$$





$$f(t)\,u(t) = \sum_{n=0}^{\infty} f_1(t-nT)$$

Let, $$f_1(t) \to F_1(s)$$

$$f_1(t-nT) \to e^{-nT_s}F_1(s)$$

$$f(t)\,u(t) \to F(s) = \sum_{n=0}^{\infty} e^{-nT_s}F_1(s)$$

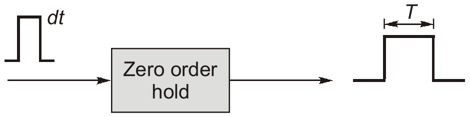
$$= \frac{F_1(s)}{1-e^{-Ts}}$$

$$f(t)u(t) \to \left[\frac{1}{1-e^{-Ts}}\right] \times \left[\begin{array}{l}\text{Transform of the } 1^{\text{st}}\\ \text{period of } f(t)\,u(t)\end{array}\right]$$

$$A(s) = \frac{1}{1-e^{-Ts}}$$

5.3 **(a)**

A zero order hold system holds the input signal value for a period of T, i.e. for an input of short duration (Δ) pulse, it produces an output pulse of duration T, the sampling period.



Input, $$x(t) = \delta(t)$$

So, $$X(s) = 1$$

Output, $$y(t) = u(t) - u(t-T)$$

So, $$Y(s) = \frac{1}{s} - \frac{e^{-Ts}}{s} = \frac{1-e^{-Ts}}{s}$$

Transfer function,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1-e^{-Ts}}{s}$$

5.4 **(a)**

The impulse response $h(t)$ of zero order hold system is

$$h(t) = u(t) - u(t-T)$$

Where T is the sampling period.

$$H(s) = \frac{1}{s} - \frac{1}{s}e^{-Ts}$$

$$H(s) = \frac{1-\exp(-Ts)}{s}$$

5.5 **(a)**

Given that, $h(t) = 4e^{-2t}$

So, $$H(s) = 4\frac{1}{(s+2)}$$

Given that

$$x(t) = u(t)$$

So, $$X(s) = \frac{1}{s}$$

We know that,

$$H(s) = \frac{Y(s)}{X(s)}$$

$$Y(s) = H(s)\,X(s)$$

$$Y(s) = \frac{4}{(s+2)} \times \frac{1}{s} = 2\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

Taking inverse Laplace transform on both side.

$$y(t) = 2[1-e^{-2t}]u(t)$$

5.6 **(c)**

In the given pole-zero pattern, poles and zeros are symmetrical about imaginary axis. This is the case of all-pass filter.

5.7 (b)

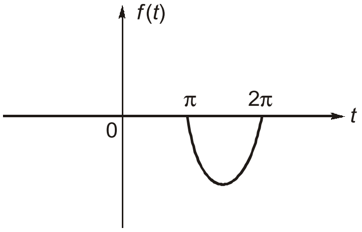
We know that,
Multiplication of two functions in frequency (s) domain is equivalent to the convolution in time domain.

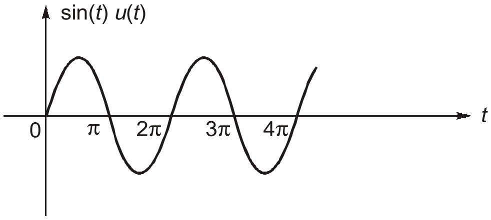
So, $v(t) = \int_0^t i(\tau) z(t-\tau) d\tau$

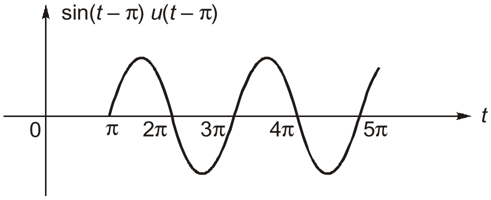
5.8 Sol.

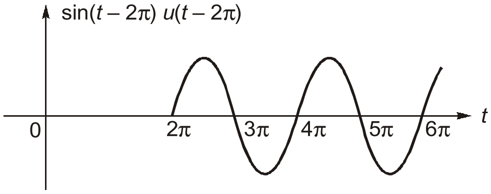
The given signal $f(t)$ is a causal periodic signal with period, $T_o = 2\pi$

$$f_1(t) = f(t) \qquad 0 < t < 2\pi$$
$$= 0 \qquad \text{otherwise}$$









So, $f_1(t) = -[\sin(t-\pi)u(t-\pi) + \sin(t-2\pi)u(t-2\pi)]$

$$\sin(t)\, u(t) \rightarrow \frac{1}{s^2+1}$$

$$\sin(t-\pi)\, u(t-\pi) \rightarrow \frac{e^{-\pi s}}{s^2+1}$$

$$\sin(t-2\pi)\, u(t-2\pi) \rightarrow \frac{e^{-2\pi s}}{s^2+1}$$

$$F_1(s) = -\left[\frac{e^{-\pi s}}{s^2+1} + \frac{e^{-2\pi s}}{s^2+1}\right]$$

$$F_1(s) = -e^{-\pi s}\left[\frac{1+e^{-\pi s}}{s^2+1}\right]$$

$$F(s) = \frac{F_1(s)}{1-e^{-T_o s}} = \frac{F_1(s)}{1-e^{-2\pi s}}$$

$$F(s) = -e^{-\pi s}\left[\frac{1+e^{-\pi s}}{s^2+1}\right] \times \frac{1}{(1-e^{-2\pi s})}$$

$$F(s) = -e^{-\pi s}\frac{(1+e^{-\pi s})}{(s^2+1)(1+e^{-\pi s})(1-e^{-\pi s})}$$

$$F(s) = \frac{-e^{-\pi s}}{(s^2+1)(1-e^{-\pi s})}$$

5.9 (d)

$$F(s) = \frac{K}{(s+1)(s^2+4)}$$

So, poles = –1, ±2*j*

Since all the poles are not in the left half of *s*-plane.

So, the final value theorem cannot be applied.

So, $\lim_{t\to\infty} f(t)$ is indeterminate.

5.10 (c)

$$r(t) \xleftrightarrow{\text{L.T.}} \frac{1}{s^2}$$

$$r(t-a) \xleftrightarrow{\text{L.T.}} e^{-as} \times \frac{1}{s^2} = \frac{e^{-as}}{s^2}$$

5.11 Sol.

If $G(s)$ is stable then all its poles must lie in the left half of *s*-plane and there is no restriction on its zeros, which can lie also in the right half of s-plane.

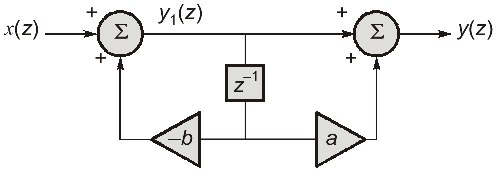
The inverse function $F(s) = \dfrac{1}{G(s)}$ may or may not be stable. The zeros of $G(s)$ may lie in the right half of *s*-plane.

Hence the given statement is not true.

So, $F(s) = \dfrac{1}{G(s)}$ is not always a stable transfer function.

6. Z-Transform

6.1 (a)



$$y_1(z) = x(z) - bz^{-1}\, y_1(z)$$

$$y_1(z)[1 + bz^{-1}] = x(z)$$

$$y_1(z) = \frac{x(z)}{(1+bz^{-1})}$$

$$y(z) = y_1(z) + az^{-1}\, y_1(z)$$

$$y(z) = y_1(z)\,[1 + az^{-1}]$$

$$y(z) = \frac{x(z)}{(1+bz^{-1})} \times (1+az^{-1})$$

Transfer function

$$H(z) = \frac{y(z)}{x(z)} = \frac{1+az^{-1}}{1+bz^{-1}}$$

6.2 (b)

Given that

$$x(n) = a^n u(n) \qquad a > 0$$

$$x(n) \xrightarrow{\text{Z.T.}} X(z) = \frac{z}{z-a};\ |z| > |a|$$

$$X(z) = \frac{1}{1-az^{-1}};\quad |z| > a$$

6.3 (a)

Given that,

Characteristic equation

$z^3 - 0.81z = 0$

$z(z^2 - 0.81) = 0$

$z(z - 0.9)\,(z + 0.9) = 0$

So, poles are $z = 0$, 0.9 and –0.9

As all the three poles are inside the unit circle, so the system is stable.

7. Sampling

7.1 (d)

$$f_s = 8000 \text{ samples/sec}$$

$$f_{m_1} = 3 \text{ kHz}$$

$$f_{m_2} = 6 \text{ kHz}$$

The spectrum of sampled signal would have

$$nf_s \pm f_m$$

So,

3 kHz, 8 ± 3, 16 ± 3, ... = 3 kHz, 5 kHz, 11 kHz, ...

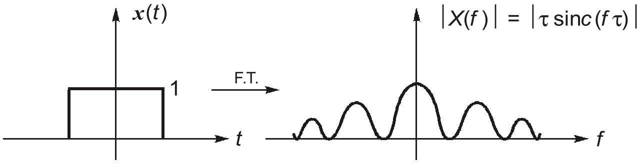
6 kHz, 8 ± 6, 16 ± 6 .... = 6 kHz, 2 kHz, 14 kHz, ...

cut-off frequencies of L.P.F. = 8 kHz

So, the filter output would have

3 kHz, 6 kHz, 2 kHz and 5 kHz

7.2 (a)



As pulse width τ is increased, the width 1/τ of the first lobe of the spectrum is decreased.

Hence, increased pulse-width in the flat-top sampling, leads to attenuation of high frequencies in reproduction.

Communication Systems

UNIT VIII

CONTENTS

VIII Communication Systems

1. Analog Communication Systems

1.1 In a superheterodyne AM receiver, the image channel selectivity is determined by
(a) The preselector and RF stages
(b) the preselector, RF and IF stages
(c) The IF stages
(d) all the stages [1987 : 2 Marks]

1.2 A carrier $A_C \cos \omega_c t$ is frequency modulated by a signal $E_m \cos \omega_m t$. The modulation index is m_f. The expression for the resulting FM signal is
(a) $A_c \cos [\omega_c t + m_f \sin \omega_m t]$
(b) $A_c \cos [\omega_c t + m_f \cos \omega_m t]$
(c) $A_c \cos [\omega_c t + 2\pi\, m_f \sin \omega_m t]$
(d) $A_c \cos\left[\omega_c t + \dfrac{2\pi m_f E_m}{\omega_m} \cos\omega_m t\right]$

[1989 : 2 Marks]

1.3 Which of the following schemes suffer(s) from the threshold effect?
(a) AM detection using envelope detection
(b) AM detection using synchronous detection
(c) FM detection using a discriminator
(d) SSB detection with synchronous detection

[1989 : 2 Marks]

1.4 A signal $x(t) = 2 \cos (\pi.10^4 t)$ volts is applied to an FM modulator with the sensitivity constant of 10 kHz/volt. Then the modulation index of the FM wave is
(a) 4 (b) 2
(c) $4/\pi$ (d) $2/\pi$

[1989 : 2 Marks]

1.5 In commercial TV transmission in India, picture and speech signals are modulated respectively as

	(Picture)		(Speech)
(a)	VSB	and	VSB
(b)	VSB	and	SSB
(c)	VSB	and	FM
(d)	FM	and	VSB

[1990 : 2 Marks]

1.6 The maximum power efficiency of an AM modulator is
(a) 25% (b) 50%
(c) 33% (d) 100%

[1992 : 2 Marks]

1.7 Which of the following demodulator(s) can be used for demodulating the signal
$x(t) = 5(1 + 2 \cos 200\, \pi t) \cos 20000\, \pi t$
(a) Envelope demodulator
(b) Square-law demodulator
(c) Synchronous demodulator
(d) None of the above

[1993 : 2 Marks]

1.8 A superheterodyne radio receiver with an intermediate frequency of 455 kHz is tuned to a station operating at 1200 kHz. The associated image frequency is ___ kHz.

[1993 : 2 Marks]

1.9 $v(t) = 5\left[\cos(10^6 \pi t) - \sin(10^3 \pi t) \times \sin(10^6 \pi t)\right]$
represents
(a) DSB suppressed carrier signal
(b) AM signal
(c) SSB upper sideband signal
(d) Narrow band FM signal

[1994 : 1 Mark]

1.10 A 10 MHz carrier is frequency modulated by a sinusoidal signal of 500 Hz, the maximum frequency deviation being 50 kHz. The bandwidth required, as given by the Carsons' rule is _____.

[1994 : 1 Mark]

1.11 Match **List-I** with **List-II** and select the correct answer using the code given below the Lists:

List-I	List-II
A. SSB	1. Envelope detector
B. AM	2. Integrate and dump
C. BPSK	3. Hilbert transform
	4. Ratio detector
	5. PLL

Codes:

	A	B	C
(a)	3	1	2
(b)	3	2	1
(c)	2	1	3
(d)	1	2	3

[1994 : 2 Marks]

2. Random Signals and Noise

2.1 The variance of a random variable X is σ_x^2. Then the variance of $-kx$ (Where k is an positive constant) is

(a) σ_x^2
(b) $-k\sigma_x^2$
(c) $k\sigma_x^2$
(d) $k^2\sigma_x^2$

[1987 : 2 Marks]

2.2 White Gaussian noise is passed through a linear narrow band filter. The probability density function of the envelope of the noise at the filter output is

(a) Uniform
(b) Poisson
(c) Gaussian
(d) Rayleigh

[1987 : 2 Marks]

2.3 In a radar receiver the antenna is connected to the receiver through a waveguide. Placing the preamplifier on the antenna side of the waveguide rather than on the receiver side leads to

(a) A reduction in the overall noise figure
(b) A reduction in interference
(c) An improvement in selectivity characteristics
(d) An improvement in directional characteristics

[1987 : 2 Marks]

2.4 Zero mean Gaussian noise of variance N is applied to a half wave rectifier. The mean squared value of the rectifier output will be

(a) Zero
(b) $N/2$
(c) $N/\sqrt{2}$
(d) N

[1989 : 2 Marks]

2.5 A part of a communication system consists of an amplifier of effective noise temperature, $T_e = 21°$ K, and a gain of 13 dB, followed by a cable with a loss of 3 dB. Assuming the ambient temperature to be 300° K, we have for this part of the communication system,

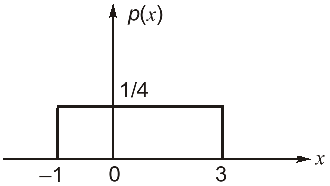
(a) effective noise temperature = 30° K
(b) effective noise temperature = 36° K
(c) noise figure = 0.49 dB
(d) noise figure = 1.61 dB

[1991 : 2 Marks]

2.6 Two resistors R_1 and R_2 (in ohms) at temperatures $T_1°$ K and $T_2°$K respectively, are connected in series. Their equivalent noise temperature is ______ °K.

[1991 : 2 Marks]

2.7 For a random variable 'X' following the probability density function, $p(x)$, shown in figure, the mean and the variance are, respectively



(a) 1/2 and 2/3
(b) 1 and 4/3
(c) 1 and 2/3
(d) 2 and 4/3

[1992 : 2 Marks]

3. Digital Communication Systems

3.1 Companding in PCM systems lead to improved signal to quantization noise ratio. This improvement is for

(a) lower frequency components only
(b) higher frequency components only
(c) lower amplitudes only
(d) higher amplitudes only

[1987 : 2 Marks]

3.2 A signal having uniformly distributed amplitude in the interval ($-V$ to $+V$), is to be encoded using PCM with uniform quantization. The signal to quantizing noise ratio is determined by the

(a) dynamic range of the signal
(b) sampling rate
(c) number of quantizing levels
(d) power spectrum of signal

[1988 : 2 Marks]

3.3 The message bit sequence to a DPSK modulator is 1, 1, 0, 0, 1, 1. The carrier phase during the reception of the first two message bits is π, π. The carrier phase for the remaining four message bits is

(a) $\pi, \pi, 0, \pi$
(b) $0, 0, \pi, \pi$
(c) $0, \pi, \pi, \pi$
(d) $\pi, \pi, 0, 0$

[1988 : 2 Marks]

3.4 In a digital communication system, transmissions of successive bits through a noisy channel are assumed to be independent events with error probability p. The probability of at most one error in the transmission of an 8-bit sequence is

(a) $\frac{7(1-p)}{8}+\frac{p}{8}$

(b) $(1-p)^8+8p(1-p)^7$

(c) $(1-p)^8+(1-p)^7$

(d) $(1-p)^8+p(1-p)^7$ [1988 : 2 Marks]

3.5 In binary data transmission DPSK is preferred to PSK because

(a) a coherent carrier is not required to be generated at the receiver

(b) for a given energy per bit, the probability of error is less

(c) the 180° phase shifts of the carrier are unimportant

(d) more protection is provided against impulse noise

[1989 : 2 Marks]

3.6 A 4 GHz carrier is DSB-SC modulated by a low pass message signal with maximum frequency of 2 MHz. The resultant signal is to be ideally sampled. The minimum frequency of the sampling impulse train should be

(a) 4 MHz (b) 8 MHz

(c) 8 GHz (d) 8.004 GHz

[1990 : 2 Marks]

3.7 In a BPSK signal detector, the local oscillator has a fixed phase error of 20°. This phase error deteriorates the SNR at the output by a factor of

(a) cos 20° (b) $\cos^2 20°$

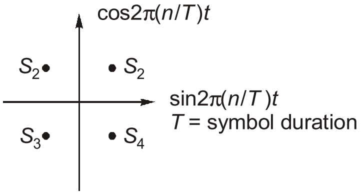
(c) cos 70° (d) $\cos^2 70°$

[1990 : 2 Marks]

3.8 A signal has frequency components from 300 Hz to 1.8 kHz. The minimum possible rate at which the signal has to be sampled is

[1991 : 2 Marks]

3.9 For the signal constellation shown in the figure, the type of modulation is



[1991 : 2 Marks]

3.10 A signal has frequency components from 300 Hz to 1.8 kHz. The minimum possible rate at which the signal has to be sampled is _______ (fill in the blank)

[1991 : 2 Marks]

3.11 The bit stream 01001 is differentially encoded using 'Delay and Ex OR' scheme for DPSK transmission. Assuming the reference bit as a '1' and assigning phases of '0' and 'π' for 1's and 0's respectively, in the encoded sequence, the transmitted phase sequence becomes

(a) π 0 π π 0 (b) 0 π π 0 0

(c) 0 π π π 0 (d) π π 0 π π

[1992 : 2 Marks]

3.12 Coherent demodulation of FSK signal can be detected using

(a) correlation receiver

(b) bandpass filters and envelope detectors

(c) matched filter

(d) discriminator detection

[1992 : 2 Marks]

3.13 Source encoding in a data communication system is done in order to

(a) enhance the information transmission

(b) bandpass filters and envelope rate detectors

(c) conserve the transmitted power

(d) discriminator detection [1992 : 2 Marks]

3.14 Sketch the waveform (with properly marked axes) at the output of a matched filter matched for a signal $s(t)$, of duration T, given by

$$s(t)=\begin{cases} A & \text{for} \quad 0 \le t < \frac{2}{3}T \\ 0 & \text{for} \quad \frac{2}{3}T \le t < T \end{cases}$$

[1993 : 2 Marks]

3.15 Increased pulse width in the flat top sampling, leads to

(a) attenuation of high frequencies in reproduction

(b) attenuation of low frequencies in reproduction

(c) greater aliasing errors in reproduction

(d) no harmful effects in reproduction

[1994 : 1 Mark]

3.16 The bandwidth required for the transmission of a PCM signal increases by a factor of ____ when the number of quantization levels is increased from 4 to 64.

[1994 : 1 Mark]

4. Information Theory and Coding

4.1 A source produces 4 symbols with probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ and $\frac{1}{8}$. For this source, a practical coding scheme has an average codeword length of 2 bits/symbols. The efficiency of the code is

(a) 1 (b) 7/8
(c) 1/2 (d) 1/4

[1989 : 2 Marks]

4.2 An image uses 512 × 512 picture elements. Each of the picture elements can take any of the 8 distinguishable intensity levels. The maximum entropy in the above image will be

(a) 2097152 bits (b) 786432 bits
(c) 648 bits (d) 144 bits

[1990 : 2 Marks]

■■■

Answers Communication Systems

1.1	(a)	1.2	(a)	1.3	(c)	1.4	(a)	1.5	(c)	1.6	(b)	1.7	(c)
1.8	(2110)	1.9	(d)	1.10	(101)	1.11	(a)	2.1	(d)	2.2	(d)	2.3	(a)
2.4	(b)	2.5	(b, c)	2.6	(sol.)	2.7	(b)	3.1	(c)	3.2	(c)	3.3	(c)
3.4	(b)	3.5	(a)	3.6	(b)	3.7	(b)	3.8	(3600)	3.9	(sol.)	3.10	(3600)
3.11	(c)	3.12	(a)	3.13	(a)	3.14	(sol.)	3.15	(a)	3.16	(3)	4.1	(b)
4.2	(b)												

Explanations Communication Systems

1. Analog Communication Systems

1.1 (a)

The image rejection should be achieved before IF stage because ones it enters into IF amplifier it becomes impossible to remove it from wanted signal. So image channel selectivity depends upon preselector and RF amplifiers only. The IF amplifiers helps in rejection of adjacent channel frequency and not image frequency.

1.2 (a)

$$X_{Fm}(t) = A_c \cos\left[\omega_c t + k_f \int m(t)\,dt\right]$$

$$X_{Fm}(t) = A_c \cos\left[\omega_c t + k_f \int E_m \cos\omega_m t\,dt\right]$$

$$X_{Fm}(t) = A_c \cos\left[\omega_c t + \frac{k_f E_m}{\omega_m} \sin\omega_m t\right]$$

Modulation index

$$m_f = \frac{k_f E_m}{\omega_m}$$

$$\boxed{X_{FM}(t) = [A_c \cos[\omega_c t + m_f \sin\omega_m t]]}$$

1.3 (c)

FM detection using a discriminator suffers from the threshold effect.

1.4 (a)

$$\boxed{\text{Modulation index} = M_f = \frac{k_f A_m}{\omega_m} = \frac{\Delta\omega}{\omega_m}}$$

$$M_f = \frac{(2\pi \times 10 \times 10^3) \times (2)}{\pi \times 10^4} = 4$$

1.5 (c)

In commercial TV transmission in India, picture signal is modulated using VSB modulation and speech or audio signal is modulated using FM modulation.

1.6 (b)

When the message signal is symmetrical square wave, 50% power efficiency can be achieved by an AM modulator.

1.7 (c)

Given that

$x(t) = 5(1 + 2\cos 200\pi t)\cos 20000\pi t$ ...(i)

The standard equation for AM signal is

$$\boxed{X_{AM}(t) = A_c(1 + m\cos\omega_m t)\cos\omega_c t}$$

By comparing the equation (i) and equation (ii), we have $m = 2$

Since the modulation index is more than 1 here, so it is the case of over modulation. When the modulation index of AM wave is more than 1 (over modulation) then the detection is possible only with synchronous modulator only. Such signals can not be detected with envelope detector.

1.8 (2110)

$$\boxed{f_{si} = f_s + 2IF}$$

$$f_{si} = 1200 + 2(455)$$

$$f_{si} = 2110 \text{ kHz}$$

1.9 (d)

$$v(t) = 5\cos(10^6\pi t) - \frac{5}{2}\cos(10^6 - 10^3)\pi t + \frac{5}{2}(10^6 + 10^3)\pi t$$

So, carrier and upper side-band are in phase and lower side band is out of phase with carrier and upper side-band.

So, the given signal is narrow band FM signal.

1.10 (101)

By Carson's rule

$$\text{BW} = 2(\Delta f + f_m)$$

$$\text{BW} = 2(50 + 0.5)$$

$$\text{BW} = 101 \text{ kHz}$$

1.11 (a)

SSB → Hilbert transform
AM → Envelope detector
BPSK → Integrate and dump

2. Random Signals and Noise

2.1 (d)

$$\text{Var}(-Kx) = E\left[(-Kx)^2\right] - (E[-Kx])^2$$

$$\sigma^2 = E[K^2x^2] - K^2E[Kx]$$

$$\sigma^2 = K^2(E[x^2] - E[x]^2)$$

$$\sigma^2 = K^2\sigma_x^2$$

2.2 (d)

Narrow band representation of noise is

$$n(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

Its envelope is $R(t) = \sqrt{n_c^2(t) + n_s^2(t)}$

Where $n_c(t)$ and $n_s(t)$ are two independent, zero mean Gaussian processes, with same variance. The resulting envelope is Rayleigh variable.

2.3 (a)

A pre amplifier is a very large gain amplifier with low noise figure.

Noise figure of cascaded amplifier can be given as:

$$\boxed{F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \cdots + \frac{F_n - 1}{G_1G_2G_3\cdots G_{n-1}}}$$

Therefore placing the pre amplifier on the antenna side of the waveguide will result in the reduction of overall noise figure of the system.

2.4 (b)

Half wave rectification can be represented as

$$y = x \quad \text{for } x \geq 0$$

$$= 0 \quad \text{for } x < 0$$

$$\text{So, } f(y) = \frac{1}{2}\delta(y) + \frac{1}{\sqrt{2\pi N}}e^{-y^2/2N}$$

$$E[Y^2] = \int_0^\infty y^2 f(y)dy$$

$$E[Y^2] = \int_0^\infty y^2\left[\frac{1}{2}\delta(y) + \frac{1}{\sqrt{2\pi N}}e^{-y^2/2N}\right]dy$$

$$E[Y^2] = 0 + \int_0^\infty \frac{y^2}{\sqrt{2\pi N}}e^{-y^2/2N}\,dy$$

$$\text{Let, } \frac{y}{\sqrt{N}} = t$$

$$dy = \sqrt{N}\,dt$$

$$E[Y^2] = \frac{1}{\sqrt{2\pi N}}\int_0^\infty Nt^2e^{-t^2/2}\sqrt{N}\,dt$$

$$= \frac{N}{\sqrt{2\pi}}\int_0^\infty t^2e^{-t^2/2}dt$$

$$E[Y^2] = \frac{N}{2}$$

2.5 (b) & (c)

Given that

$$T_e = 21°\text{K}$$

$$T_a = 300°\text{K}$$

$$\text{Gain} = (G)_{\text{dB}} = 13$$

$$13 = 10\log_{10}(G)$$

$$G = 19.95$$

$$\text{Cable loss} = (3)_{\text{dB}} = 10\log_{10}(L)$$

$$L = 1.995$$

For a cable,

$$\text{Noise figure} = F_2 = \text{cable loss} = L$$

$$F_2 = 1.995$$

$$\text{Noise figure of amplifier} = F_1 = 1 + \frac{T_e}{T_a}$$

$$F_1 = 1 + \frac{21}{300} = 1.07$$

Noise figure of cascaded amplifier

$$F = F_1 + \frac{F_2 - 1}{G}$$

$$= 1.07 + \frac{1.995 - 1}{19.95} = 1.12$$

$$F = 0.49 \text{ dB}$$

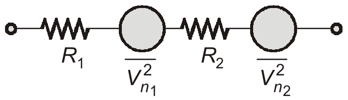
T_e' of the cascaded amplifier.

$$T_e' = (F - 1)T_a$$

$$T_e' = (1.12 - 1)300$$

$$T_e' = 36°\text{K}$$

2.6 Sol.



$$\overline{V_n^2} = \overline{V_{n_1}^2} + \overline{V_{n_2}^2}$$

$$\overline{V_n^2} = 4KT_e\,BR = 4KT_eB(R_1 + R_2)$$

$$\overline{V_{n_1}^2} = 4KT_1BR_1$$

$$\overline{V_{n_2}^2} = 4KT_2BR_2$$

So,

$$4K\,T_eB(R_1 + R_2) = 4K\,T_1BR_1 + 4KT_2BR_2$$

$$4KT_eB(R_1 + R_2) = 4KB(R_1T_1 + R_2T_2)$$

$$Te(R_1 + R_2) = R_1T_1 + R_2T_2$$

$$T_e = \frac{R_1T_1 + R_2T_2}{R_1 + R_2}$$

2.7 (b)

$$\text{Mean} = \mu_X = E[X] = \int_{-\infty}^{\infty} xp(x)dx$$

$$\mu_X = \int_{-1}^{3} x\frac{1}{4}dx = \frac{1}{4}\left[\frac{x^2}{2}\right]_{-1}^{3}$$

$$\mu_X = \frac{1}{4}\times\frac{1}{2}\left[9 - (-1)^2\right] = \frac{1}{8}\times 8$$

$$\mu_X = 1$$

$$\text{Variance} = \sigma_X^2 = E\left[(x - \mu_X)^2\right] = \int_{-\infty}^{\infty}(x - \mu_X)^2 p_X(x)dx$$

$$\sigma_X^2 = \int_{-1}^{3}(x - 1)^2\frac{1}{4}dx = \frac{1}{4}\int_{-1}^{3}(x - 1)^2 dx$$

$$\sigma_X^2 = \frac{1}{4}\int_{-1}^{3}(x^2 + 1 - 2x)dx$$

$$\sigma_X^2 = \frac{1}{4}\left[\frac{x^3}{3} + x - \frac{2x^2}{2}\right]_{-1}^{3}$$

$$\sigma_X^2 = \frac{1}{4}\left[\left(\frac{27}{3} + 3 - 9\right) - \left(-\frac{1}{3} - 1 - 1\right)\right]$$

$$\sigma_X^2 = \frac{1}{4}\left[3 + \frac{1}{3} + 2\right] = \frac{1}{4}\times\frac{1}{3}[9 + 1 + 6]$$

$$\sigma_X^2 = \frac{4}{3}$$

3. Digital Communication Systems

3.1 (c)

Companding results in making SNR uniform, throughout the signal, irrespective of amplitude levels. Since, in uniform quantization, step size is same, the quantization noise power is uniform, throughout the signal.

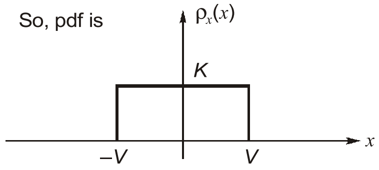
Thus, higher amplitudes of signal, will have better SNR than the lower amplitudes.

Hence, companding is used for improving SNR at lower amplitudes.

3.2 (c)

Since the signal is uniformly distributed in the interval –V to +V.

So, pdf is



Area under pdf is unity

$$K[V-(-V)] = 1$$

$$2\,VK = 1$$

$$K = \frac{1}{2\,V}$$

So, $\rho_X(x) = \frac{1}{2\,V}$ $\quad -V$ to V

$= 0$ $\quad$ otherwise

$$\text{Signal power} = s = \int_{-\infty}^{\infty} x^2 \rho_x(x)\,dx$$

$$s = \int_{-V}^{V} x^2 \frac{1}{2\,V} dx = \frac{1}{2\,V}\left[\frac{x^3}{3}\right]_{-V}^{V}$$

$$s = \frac{1}{2\,V}\left[\frac{V^3}{3} - \frac{(-V^3)}{3}\right] = \frac{1}{2\,V}\cdot\frac{2\,V^3}{3} = \frac{V^2}{3}$$

In uniform quantization,

$$\text{Quantization noise power} = \text{QNP} = \frac{\Delta^2}{12}$$

$$\text{where, step size} = \Delta = \frac{V_{p-p}}{L} = \frac{V_{p-p}}{2^n}$$

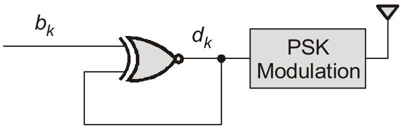
$$\text{QNP} = \frac{V_{p-p}^2}{12\times 2^{2n}} = \frac{(2V)^2}{12\times 2^{2n}} = \frac{V^2}{3\times 2^{2n}}$$

$$\text{SQNR} = \frac{S}{\text{QNP}} = \frac{V^2}{3}\times\frac{3\times 2^{2n}}{V^2} = 2^{2n}$$

So, SQNR $\propto 2^{2n}$

So, signal to quantizing noise ratio is determined by the number of quantizing levels.

3.3 (c)



EX-NOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$Y = A \odot B$

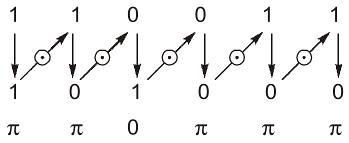
$Y = AB + \bar{A}\bar{B}$

0 → is represented with a carrier has phase 'π'

1 → is represented with a carrier has phase '0'

$d_k = b_X \oplus d_{k-1}$ or $d_k = b_k \odot d_{k-1}$

Logic function here is EX-NOR.



Hence, the answer is 0, π, π, π.

3.4 (b)

Let getting an error be success.

$P(\text{success}) = P$

$P(\text{failure}) = 1 - P$

$P(x = \text{at most } 1) = P(x = 0) + P(x = 1)$

$= 8C_0(P)^0 * (1-P)^{8-0} + {}^8C_1(P)^1\,(1-P)^{8-1}$

$= (1-P)^8 + 8P(1-P)^7$

3.5 (a)

A coherent carrier is not required to be generated at the receiver.

3.6 (b)

$$f_c = 4\text{ GHz} = 4000\text{ MHz}$$

$$f_m = 2\text{ MHz}$$

$$f_H = f_c + f_m = 4000 + 2 = 4002\text{ MHz}$$

$$f_L = f_c - f_m = 4000 - 2 = 3998\text{ MHz}$$

$$f_s = \frac{2f_H}{K}$$

$$K = \left\lfloor \frac{f_H}{f_H - f_L} \right\rfloor = \left\lfloor \frac{4002\text{ MHz}}{4\text{ MHz}} \right\rfloor = \lfloor 1000.5 \rfloor = 1000$$

$$f_s = \frac{2\times 4002\text{ MHz}}{1000} = 8.004\text{ MHz} \simeq 8\text{ MHz}$$

3.7 (b)

In BPSK if detector has a fixed phase error ϕ then the output power would change by a factor $\cos^2\phi$.

3.8 (3600)

3600 samples/sec

Given that

$$f_H = 1800\text{ Hz}$$

$$f_L = 300\text{ Hz}$$

So, $\boxed{\text{B.W.} = f_H - f_L}$

$$\text{B.W.} = 1800 - 300 = 1500\text{ Hz}$$

$$\boxed{K = \frac{f_H}{\text{B.W.}}}$$

$$K = \frac{1800}{1500}$$

So, $K = 1$

$$\boxed{(f_s)_{\min} = \frac{2f_H}{K}}$$

$$(f_s)_{\min} = \frac{2\times 1800}{1} = 3600\text{ samples/sec}$$

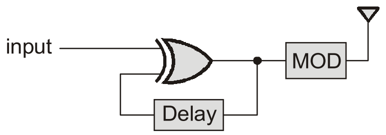
3.9 Sol.

For different phases are given which are 90° apart with adjacent signal. So, it is the clear of QPSK (Quadrature Phase Shift Keying).

3.10 (3600)

$$f_s = 3600 \text{ samples/sec}$$
$$f_H = 1800 \text{ Hz}$$
$$f_L = 300 \text{ Hz}$$
$$\text{B.W.} = f_H - f_L = 1800 - 300 = 1500 \text{ Hz}$$
$$K = \frac{f_H}{\text{B.W.}} = \frac{1800}{1500} = 1$$
$$f_{s(\text{min})} = \frac{2f_H}{K} = \frac{2 \times 1800}{1} = 3600 \text{ samples/ sec}$$

3.11 (c)



EX-XOR

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

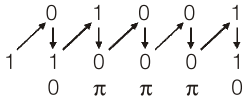
$$Y = A \oplus B$$
$$Y = A\bar{B} + \bar{A}B$$

Given that

reference bit = 1

logic 0 → π

logic 1 → 0°



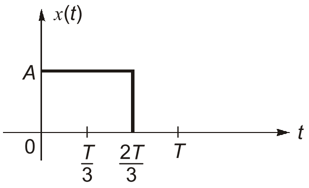
3.12 (a)

Coherent demodulation of FSK signal can be detected using correlation receiver.

3.13 (a)

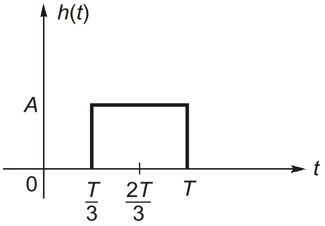
The purpose of source encoding in a data communication system is to represent non-electrical discrete symbols with binary codes and thus enhance the information transmission and purpose of channel encoding is to decrease the probability of error. The channel coding helps in detection and correction of errors.

3.14 Sol.



We know that impulse-response of a matched filer is

$$h(t) = x(T-t)$$



The output of matched filter

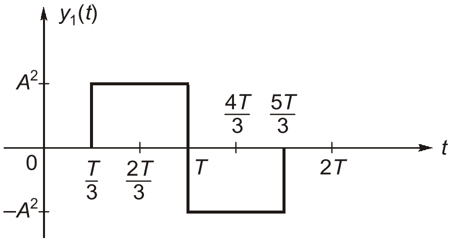
$$y(t) = x(t) * h(t)$$

$$\frac{dh}{dt}(t) = A\delta\left(t - \frac{T}{3}\right) - A\delta(t - T)$$

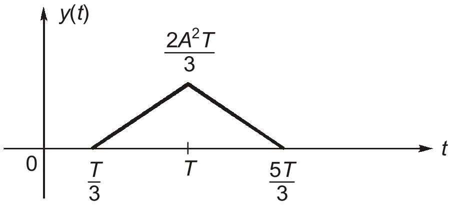
$$y(t) = \int_{-\infty}^{t} x(t) * A\delta\left(t - \frac{T}{3}\right) - x(t) * A\delta(t - T)\,dt$$

$$y(t) = \int_{-\infty}^{t} A\left[x\left(t - \frac{T}{3}\right) - x(t - T)\right]dt$$

$$= \int_{-\infty}^{t} y_1(t)\,dt$$



$$y(t) = \int_{-\infty}^{t} y_1(t)\,dt$$



3.15 (a)

Increased pulse width in the flat-top sampling leads to greater attenuation of high frequencies in reproduction. This effect is known as aperture effect.

3.16 (3)

$$\boxed{(\text{B.W.})_{\text{PCM}} = nfs}$$

$$\boxed{n = \log_2 L}$$

$$n_1 = \log_2 4 = 2$$
$$n_2 = \log_2 64 = 6$$

$$(\text{B.W.})_1 = n_1 fs = 2fs$$

$$(\text{B.W.})_2 = n_2 fs = 6fs$$

$$\frac{(\text{B.W.})_2}{(\text{B.W.})_1} = \frac{6fs}{2fs} = 3 \text{ times}$$

$$(\text{B.W.})_2 = 3(\text{B.W.})_1$$

4. Information Theory and Coding

4.1 (b)

$$\text{Entropy} = H = -\sum_{i=1}^{n} P_i \log_2(P_i)$$

$$H = -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{8}\log_2\left(\frac{1}{8}\right) + \frac{1}{8}\log_2\left(\frac{1}{8}\right)\right]$$

$$H = \frac{1}{2} + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{8}\times 3$$

$$H = 1 + \frac{3}{4} = \frac{7}{4}$$

$$\text{Code efficiency} = \eta = \frac{H}{L}$$

$$\eta = \frac{7/4}{2} = \frac{7}{8}$$

4.2 (b)

$$n = \log_2 L = \log_2 8 = 3$$

$$\text{Maximum entropy} = 512 \times 512 \times n$$

$$= 512 \times 512 \times 3 = 786432 \text{ bits/image}$$

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Engineering Mathematics

UNIT IX

CONTENTS

IX

Engineering Mathematics

1. Linear Algebra

1.1 The rank of $(m \times n)$ matrix (where $m < n$) cannot be more than

(a) m (b) n

(c) mn (d) none

[1994 : 1 Mark]

1.2 Solve the following system of equations

$x_1 + x_2 + x_3 = 3$

$x_1 - x_2 = 0$

$x_1 - x_2 + x_3 = 1$

(a) Unique solution

(b) No solution

(c) Infinite number of solutions

(d) Only one solution [1994 : 2 Marks]

2. Differential Equations

2.1 $y = e^{-2x}$ is a solution of the differential equation $y'' + y' - 2y = 0$. (True/False).

[1994 : 1 Mark]

■■■

Answers Differential Equations

1.1 (a) 1.2 (a) 2.1 (True)

Explanations Differential Equations

1. Linear Algebra

1.1 (a)

If A is a matrix of order $m \times n$ $(m < n)$ then rank of A is $\le \min\{m, n\}$

$$\therefore \quad r(A) \ngtr m$$

1.2 (a)

Augmented matrix (AB) =

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & -1 & 0 & 0 \\ 1 & -1 & 1 & 1 \end{bmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & -1 & -3 \\ 0 & -2 & 0 & -2 \end{bmatrix} R_3 - R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & -1 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad \rho(A) = \rho(AB) = 3$$

$\Rightarrow$ Unique solution.

2. Differential Equations

2.1 (True)

y + y¢ – 2y = 0

$$(D^2 + D - 2) = 0$$

Auxiliary equation is

$$m^2 + m - 2 = 0$$

$$(m + 2)(m - 1) = 0$$

$$m = -2, 1$$

complimentary function is $C_1 e^{-2x} + C_2 e^x$

$\therefore$ solution is $y = C_1 e^{-2x} + C_2 e^x$

$\therefore$ $y = e^{-2x}$ is a solution of $y'' + y' - 2y = 0$

■■■